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The Council elected the following as Fellows of the Society at the Council meeting on 1941 February 14:—

Norman Douglas Anderson, Scindia Steam Navigation Co., Bombay, India;
Brian Percival Mullins, B.Sc., 230 Broadwalk, Blackheath, London, S.E. 3;
Clement Frederick Press, F.M.S. Railways, Port Swettenham, Selangor, Federated Malay States;
Hyman S. Spigl, Government Astronomer of Western Australia, Perth Observatory, W.A.; and
Gordon W. Wares, Brenau College, Gainesville, Georgia, U.S.A.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Ronald Frank Newby, 79 St. John's Grove, Harrogate, Yorks (proposed by Violet F. White); and

Chandrakant Gajanan Pendse, B.Sc., M.A., Ph.D., Colaba Observatory, Bombay, India (proposed by W. M. Smart).

CONVECTION IN THE OUTER LAYERS OF THE SUN

R. v. d. R. Woolley

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1. A discussion of the history of the investigations into the nature of the transport of energy in the stars is given in Eddington's *Internal Constitution of the Stars*, sections 68-70. It was realised at an early date that conduction was inadequate and it was supposed that convection was the effective agent. In 1893 Sampson felt dissatisfied with the existing theory of convective equilibrium and constructed a theory of radiative equilibrium, and in 1905 Schwarzschild demonstrated that convection currents are thermodynamically unstable in the presence of radiative equilibrium if the ratio of the specific heats of the material exceeds $4/3$. Since this ratio is $5/3$ for a monatomic gas, convection was considered to be impossible in a stellar atmosphere. Further, the successes which attended the application of the theory of radiative equilibrium to the outer layers of the Sun, especially at the hands of E. A. Milne, strengthened the view that convection did not occur.

However, in 1930 Unsöld* called attention to the fact that the ratio of the specific heats of a gas dissociating into ions and electrons can fall below the value $4/3$. Unsöld calculated that the ratio of the specific heats of solar material fell below the critical value in a certain zone, the convection zone, roughly the zone in which the ionisation of hydrogen is appreciable without being nearly complete. This idea was followed up by Siedentopf† who published several papers. In 1936 H. H. Plaskett‡ discussed the nature of solar granulation in an important and suggestive paper. He pointed out afresh that observations of darkening of the limb should reveal any departure from the normal radiative temperature gradient in the solar atmosphere. Plaskett analysed a particular set of observations of solar limb darkening at 5500 Å. by Moll, Burger and van der Bilt and found a temperature gradient differing from the Milne (radiative equilibrium) distribution.

Against Plaskett's result must be set Milne's analysis§ of Abbot's observations, which are much more extensive than those of the Dutch observers. Milne obtained good agreement between the observations of Abbot in thirteen wave-lengths from 3250 Å. to 12,250 Å. and the predictions of his theory of radiative equilibrium. It is worth remarking that Milne did not get the same good agreement with Abbot's observations when he integrated the results for white light, and that Milne himself mentioned|| the possibility of attributing the discrepancy to convection.

In the present paper an attempt is made to examine in particular the velocities required to bring about an improvement in the theoretical figures for the darkening of the limb. It is found that the velocities required would blur the Fraunhofer lines. Our main conclusion is that convection must occur too deep to affect the darkening of the solar limb.

2. In the remainder of this paper we shall assume explicitly that conduction can be ignored and that the only available means of transporting energy are radiation and convection. If we assume that the atmosphere is characterised by a single temperature at any optical depth (an assumption which we shall soon discard) we have the Schwarzs-

* *Zs. f. Ap.*, 1, 138, 1930.

† *A.N.*, 247, 297, 1933; 249, 53, 1933; 255, 157, 1935.

‡ *M.N.*, 96, 402, 1936.

§ *Phil. Trans.*, 223, 201, 1922.

|| *Loc. cit.*, paragraph 5.

child theorem that the adiabatic temperature gradient $\left| \left(\frac{dT}{dx} \right)_s \right|$ is less than the radiative gradient $\left| \left(\frac{dT}{dx} \right)_M \right|$. The actual temperature gradient must exceed the adiabatic gradient in order that convection currents may transport a net flux of energy outwards. Further, the actual temperature gradient must be equal to or less than the radiative gradient, because if the actual gradient exceeds the radiative gradient at any point, radiation will transport at that point a net flux of energy greater than the surface net flux, which is impossible unless there is a sink of energy in the outer layers of a star, which we cannot admit. Hence the actual temperature gradient must satisfy the inequality

$$\left| \left(\frac{dT}{dx} \right)_s \right| < \left| \frac{dT}{dx} \right| \leq \left| \left(\frac{dT}{dx} \right)_M \right|.$$

3. Consider the radiation field in a region in which convection may occur. Following Milne's classical treatment with slight modifications, let $I_\nu(\theta)d\nu d\omega/4\pi$ be the flow per sq. cm. per sec. of radiation in frequencies between ν and $\nu + d\nu$ travelling in a solid angle $d\omega$ at an angle θ to the vertical (which is the axis of x , positive outwards). Consider a cylinder of unit cross-section and of length $ds = dx \sec \theta$ with its axis in the direction θ . Let ρ be the density of the atmosphere. The coefficient of absorption is k_ν if the amount of radiation absorbed in the cylinder from the flow $I_\nu(\theta)$ is

$$k_\nu \rho ds \cdot I_\nu(\theta) d\nu d\omega / 4\pi.$$

Further, let j_ν be the emission of radiation per gramme per second, supposed isotropic so that $j_\nu d\omega/4\pi$ will go into the solid angle $d\omega$. Then

$$\cos \theta \frac{dI_\nu(\theta)}{\rho dx} = -k_\nu I_\nu(\theta) + j_\nu. \quad (3.1)$$

Now let

$$\frac{1}{4\pi} \int I_\nu(\theta) d\omega = \mathcal{J}_\nu, \quad \frac{1}{4\pi} \int I_\nu(\theta) \cos \theta d\omega = H_\nu \quad \text{and} \quad \frac{1}{4\pi} \int I_\nu(\theta) \cos^2 \theta d\omega = K_\nu.$$

Multiply (3.1) by $d\omega/4\pi$ and integrate over a sphere, and then multiply (3.1) by $\cos \theta d\omega/4\pi$ and integrate over a sphere. We get

$$\left. \begin{aligned} \frac{dH_\nu}{\rho dx} &= -k_\nu \mathcal{J}_\nu + j_\nu, \\ \frac{dK_\nu}{\rho dx} &= -k_\nu H_\nu. \end{aligned} \right\} \quad (3.2)$$

Now write

$$\int \mathcal{J}_\nu d\nu = \mathcal{J}, \quad \int H_\nu d\nu = H \quad \text{and} \quad \int K_\nu d\nu = K.$$

Further, let

$$\bar{k} = H^{-1} \int k_\nu H_\nu d\nu \quad \text{and let} \quad d\tau = -\bar{k} \rho dx.$$

Then equations (3.2) become

$$\left. \begin{aligned} \frac{dH}{d\tau} &= \frac{1}{\bar{k}} \left[\int k_\nu \mathcal{J}_\nu d\nu - j/\bar{k} \right], \\ \frac{dK}{d\tau} &= H. \end{aligned} \right\} \quad (3.3)$$

Adopting the well-known approximation for the θ distribution of $I(\theta)$, namely, $K = \frac{1}{3}\mathcal{F}$, we get

$$\frac{d\mathcal{F}}{d\tau} = 3H. \quad (3.4)$$

Where there is no transport of heat by convection H is constant with τ and (3.4) integrates to

$$\mathcal{F} = 3H\tau + \text{constant},$$

or with the standard boundary condition $\mathcal{F} = 2H$ at $\tau = 0$,

$$\mathcal{F} = H(2 + 3\tau),$$

the result for radiative equilibrium. But let us suppose that convection transports a fraction $\beta(\tau)$ of the energy flux. Then

$$H = H_0(1 - \beta(\tau))$$

where H_0 is the surface flux and $0 \leq \beta(\tau) < 1$. Let $\phi(\tau) = \int_0^\tau \beta(\tau) d\tau$. Then since

$\frac{d\mathcal{F}}{d\tau} = 3H_0(1 - \beta(\tau))$ we have

$$\mathcal{F} = H_0(2 + 3\tau - 3\phi(\tau)). \quad (3.5)$$

We shall now investigate Milne's standard case of "gray" material, namely, the case where k_ν is independent of ν and equal to k . Then from (3.3) we have

$$j = k \left(\mathcal{F} - \frac{dH}{d\tau} \right),$$

so that

$$j = kH_0(2 + 3\tau - 3\phi(\tau) + \beta'(\tau)). \quad (3.6)$$

Equation (3.6) leads to an expression for the darkening of the limb in integrated light, because a cylinder of length $ds = dx \sec \theta$ will emit the amount $j\rho dx \sec \theta d\omega / 4\pi$ into a pencil $d\omega$ inclined at an angle θ to the vertical. This will be reduced to

$$j\rho dx \sec \theta \frac{d\omega}{4\pi} e^{-k\rho x \sec \theta} \quad (3.7)$$

on emergence at the surface. Hence the intensity of radiation leaving the surface at an angle θ is obtained by integrating (3.7) with respect to x from 0 to $-\infty$, or

$$\begin{aligned} I(0, \theta) &= \int_0^{-\infty} j\rho dx \sec \theta e^{-k\rho x \sec \theta} \\ &= H_0 \left\{ 2 + 3 \cos \theta + \int_0^\infty (\beta'(\tau) - 3\phi(\tau)) e^{-\tau \sec \theta} d\tau \sec \theta \right\}. \end{aligned} \quad (3.8)$$

Following Unsöld, let us suppose that the convection zone starts at some small optical depth and continues to an optical depth so large that no radiation from below the zone will contribute effectively to the integral in (3.8), and let us suppose for the moment that a constant proportion of the energy flux is carried by convection. As a model we may adopt

Range	$\beta(\tau)$	$\phi(\tau)$
$0 < \tau < \tau_1$	0	0
$\tau_1 < \tau < \tau_2$	$\frac{1}{2}\beta(1 - \cos p(\tau - \tau_1))$	$\frac{1}{2}\beta(\tau - \tau_1) - \frac{1}{2}\beta p^{-1} \sin p(\tau - \tau_1)$
$\tau_2 < \tau < \infty$	β	$\beta(\tau - \frac{1}{2}(\tau_1 + \tau_2))$

where β is constant and $p(\tau_2 - \tau_1) = \pi$. The continuity of $\beta'(\tau)$ is obtained by the use of this model. Integrating (3.8) we get

$$I(0, \theta) = H_0 \left\{ 2 + 3 \cos \theta + \frac{p^2 \beta \cos \theta}{2 + 2p^2 \cos^2 \theta} (1 - 3 \cos^2 \theta) (e^{-\tau_1 \sec \theta} + e^{-\tau_2 \sec \theta}) \right\}. \quad (3.81)$$

Let $\tau_2 \rightarrow \tau_1$ so that p becomes large. (This gives an abrupt boundary between radiation zone and convection zone.) Then

$$I(0, \theta) = H_0 \{ 2 + 3 \cos \theta + \beta \sec \theta (1 - 3 \cos^2 \theta) e^{-\tau_1 \sec \theta} \}. \quad (3.9)$$

This reduces to Milne's result for "gray" material (k_ν independent of ν) if we put $\beta = 0$ or $\tau_1 \rightarrow \infty$, Milne's result being

$$I(0, \theta) = H_0 (2 + 3 \cos \theta).$$

Equation (3.9) may be fitted to Milne's colour integration of Abbot's observations. A good fit is obtained by setting $\tau_1 = 1.25$ and $\beta = 0.30$. Table I shows a comparison between equation (3.9) with these values and the observations, the result for radiative equilibrium $I(0, \theta) = H_0 (2 + 3 \cos \theta)$ being shown as well.

TABLE I
Convection and Observed Darkening of the Limb

$\cos \theta$	1.0000	0.9165	0.8352	0.7599	0.6614	0.5651	0.4841	0.3919	0.3122
Radiative Equilibrium	1000	950	901	856	797	739	690	635	587
Observed	1000	955	912	871	822	769	722	665	612
Convection (Eqn. (3.9))	1000	957	915	875	821	766	718	661	611

The improvement between theory and observation brought about by the introduction of the convection terms is striking. Although there are two disposable constants, they have quite reasonable values, and if we knew that the observations were reliable to the figures shown, *and if we knew that the material was "gray,"* we should infer the existence of the convection from the observations. The evidence of the later sections of this paper (especially paragraph 8) will, however, show that this value of β so close to the surface is quite impossible on solid thermodynamical grounds and also on the very firm ground that it necessitates velocities for the convection currents which would blur the Fraunhofer lines. Mention must be made of the effect of the Fraunhofer lines on the darkening of the limb. Professor Milne published in 1928 a note * indicating that the high absorption coefficients in the line frequencies in otherwise "gray" material should decrease the theoretical darkening of the limb, and so improve the agreement between the theoretical values for "gray" material in radiative equilibrium with Abbot's (integrated) observations. The present writer is unable to repeat this conclusion, and a brief discussion will be found as an appendix to this paper.

4. In an atmosphere in which convection takes place there must be upward and downward moving matter. We shall suppose that all the matter is moving one way or the other. The upward matter must have a higher temperature than the downward moving matter: we suppose that at any given optical depth all the upward moving matter has the same temperature T_1 , density ρ_1 , and velocity ξ_1 , the corresponding quantities for downward moving matter having the suffix 2. At any depth let the upward moving matter occupy the fraction a_1 of the surface area and let $a_2 = 1 - a_1$. The condition that there is no secular transport of mass across any surface gives

$$a_1 \rho_1 \xi_1 + a_2 \rho_2 \xi_2 = 0.$$

* *Observatory*, 51, 88, 1928.

The density of the atmosphere is $\rho = \alpha_1 \rho_1 + \alpha_2 \rho_2$. We shall assume that the coefficient of absorption k is the same for both upward and downward streams. We may define the independent variable τ in any way we please, and set $d\tau = -k\rho dx$.

Assuming that both upward and downward streams are aggregated in such large masses that they are internally in radiative equilibrium, the net flux of radiation in the upward and downward matter is

$$\frac{1}{3} \frac{\rho}{\rho_1} \frac{dB(T_1)}{d\tau} \quad \text{and} \quad \frac{1}{3} \frac{\rho}{\rho_2} \frac{dB(T_2)}{d\tau}$$

respectively. The average flow over a surface large enough to sample both kinds of motion is $H_0(1 - \beta(\tau))$ in the notation of paragraph 3. We have accordingly

$$3H_0(1 - \beta(\tau)) = \frac{\alpha_1 \rho}{\rho_1} \frac{dB(T_1)}{d\tau} + \frac{\alpha_2 \rho}{\rho_2} \frac{dB(T_2)}{d\tau}. \quad (4.1)$$

Again, the net flux carried by convection is $H_0\beta(\tau)$. Since both streams are at the same pressure,

$$\begin{aligned} H_0\beta(\tau) &= \alpha_1 \rho_1 \xi_1 \int_0^{T_1} C_p dT + \alpha_2 \rho_2 \xi_2 \int_0^{T_2} C_p dT \\ &= \alpha_1 \rho_1 \xi_1 \int_{T_2}^{T_1} C_p dT, \end{aligned} \quad (4.2)$$

where C_p is the specific heat at constant pressure.

Introduce a temperature T defined by $T^4 = \alpha_1 T_1^4 + \alpha_2 T_2^4$. Let $T_1 = T(1 + \alpha_2 \gamma)$ and suppose that γ is a small quantity so that γ^2 may be neglected. Then $T_2 = T(1 - \alpha_1 \gamma)$. The equation of state of the ionised gas, which we shall suppose for simplicity to be hydrogen, is $p = \rho \mathbf{R}(1 + y)T$, where y is the degree of ionisation and $\mathbf{R}$ the universal gas constant. Then since $\rho = \alpha_1 \rho_1 + \alpha_2 \rho_2$ and p is the same for both streams, we have

$$\begin{aligned} \frac{\rho}{\rho_1} &= 1 + \alpha_2 \gamma \left(1 + \frac{T}{1 + y} \left(\frac{\partial y}{\partial T} \right)_p \right) = 1 + \alpha_2 \gamma Q, \\ \frac{\rho}{\rho_2} &= 1 - \alpha_1 \gamma \left(1 + \frac{T}{1 + y} \left(\frac{\partial y}{\partial T} \right)_p \right) = 1 - \alpha_1 \gamma Q. \end{aligned}$$

Again

$$\frac{dB(T_1)}{d\tau} = 4acT_1^3 \frac{dT_1}{d\tau} = 4acT^3 (1 + 3\alpha_2 \gamma) \left\{ \frac{dT}{d\tau} (1 + \alpha_2 \gamma) + \alpha_2 T \frac{d\gamma}{d\tau} \right\},$$

with a similar expression for $\frac{dB(T_2)}{d\tau}$. Inserting these values in (4.1) we get

$$3H_0(1 - \beta(\tau)) = 4acT^3 \frac{dT}{d\tau} + \text{terms of order } \gamma^2. \quad (4.3)$$

Integrate with respect to τ subject to $T = T_0$ at $\tau = 0$ and using $acT_0^4 = 2H$ and $\phi(\tau) = \int_0^\tau \beta(\tau) d\tau$ we get

$$T_4 = T_0^4 (1 + \frac{3}{2}(\tau - \phi(\tau))). \quad (4.31)$$

Equation (4.3) may be written

$$T_0^4 (1 - \beta(\tau)) = \frac{8}{3} T^3 \frac{dT}{d\tau}, \quad (4.4)$$

and equation (4.2) becomes

$$acT_0^4 \beta(\tau) = 2\alpha_1 \rho_1 \xi_1 C_p \gamma T. \quad (4.5)$$

We may eliminate $\beta(\tau)$ from these equations and get

$$T_0^4 = \frac{8}{3} T^3 \frac{dT}{d\tau} + \frac{2\alpha_1 \rho_1 \xi_1}{ac} C_p \gamma T. \quad (4.6)$$

Since ξ_1 and γ are essentially positive, $\frac{dT}{d\tau} \leq \frac{3}{8} \frac{T_0^4}{T^3}$ confirming the inequality $\left| \frac{dT}{dx} \right| \leq \left| \frac{dT}{dx} \right|_M$ in paragraph 2.

5. Let K be the kinetic energy of one gramme of the gas and E its internal energy. Let one gramme of the gas pass from state 1 to state 2 doing work W_{12} and losing by conduction or radiation the amount of heat Q_{12} . Then from the conservation of energy

$$K_1 + E_1 = K_2 + E_2 + W_{12} + Q_{12}.$$

If the gas is moving parallel to the x axis with a velocity ξ then $K = \frac{1}{2} \xi^2$, and if the heat loss is q ergs per gramme per second (by radiation, conduction being negligible) we have

$$\frac{dE}{dx} + \frac{dW}{dx} + \xi \frac{d\xi}{dx} + \frac{q}{\xi} = 0. \quad (5.1)$$

Write

$$\left(\frac{\partial(E+W)}{\partial T} \right)_p = L \quad \text{and} \quad \left(\frac{\partial(E+W)}{\partial p} \right)_T = M.$$

Then

$$\frac{dE}{dx} + \frac{dW}{dx} = L \frac{dT}{dx} + M \frac{dp}{dx},$$

and dividing (5.1) by $k\xi^{-1}$ and using $d\tau = -k\rho dx$ we have

$$\xi \rho \left\{ L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau} \right\} = \frac{q}{k}. \quad (5.2)$$

To evaluate L and M , let Q be the quantity of heat taken in by and S the entropy of one gramme of the gas. Then

$$dE + dW = dQ = TdS.$$

Hence $L = \left(\frac{\partial Q}{\partial T} \right)_p = C_p$ the specific heat at constant pressure. Also

$$M = T \left(\frac{\partial S}{\partial p} \right)_T = -T \left(\frac{\partial v}{\partial T} \right)_p,$$

where v is the volume of one gramme of the gas.

The degree of ionisation y is given by Saha's formula :

$$\ln \frac{y}{1-y} P_e = \frac{5}{2} \ln T - \frac{\chi}{RT} + \text{const.},$$

where P_e is the electron pressure, χ the ionisation potential and R is Boltzmann's constant. We have from the equation of state $p = \mathbf{R}(1+y)\rho T$ the result

$$M = -\frac{\mathbf{R}T}{p} \left\{ 1-y + T \left(\frac{\partial y}{\partial T} \right)_p \right\}.$$

But from Saha's relation

$$T \left(\frac{\partial y}{\partial T} \right)_p = \frac{1}{2} y(1-y^2) \left(\frac{\chi}{RT} + \frac{5}{2} \right), \quad (5.31)$$

$$p \left(\frac{\partial y}{\partial p} \right)_T = -\frac{1}{2} y(1-y)^2. \quad (5.32)$$

The entropy S is given by

$$S = R \left\{ 2 \ln \frac{y}{1-y} + (1+y) \left(\frac{\chi}{RT} + \frac{5}{2} \right) + \text{const.} \right\}.$$

But $C_p = T \left(\frac{\partial S}{\partial T} \right)_p$ so that

$$L = C_p = R(1+y) \left\{ \frac{5}{2} + \frac{1}{2} y(1-y) \left(\frac{\chi}{RT} + \frac{5}{2} \right)^2 \right\} \quad (5.41)$$

$$M = -RTp^{-1}(1+y) \left\{ 1 + \frac{1}{2} y(1-y) \left(\frac{\chi}{RT} + \frac{5}{2} \right) \right\}. \quad (5.42)$$

We must now consider the radiative equilibrium of a moving mass. Let h be the volume of the mass divided by its surface area, and suppose that the mass is at a uniform temperature T while its surroundings are at T' . Then the heat lost by radiation to its surroundings per gramme is

$$\{B(T) - B(T')\} / \rho h.$$

The internal energy of the mass E is governed by (5.1), so that its temperature (judged by thermal motion of the particles) is fixed by this equation. We may suppose that the material is in local thermodynamical equilibrium, which only means that the local radiation balance is secured by the emission of temperature radiation. Then the radiation field will adjust itself to be equal in energy per c.c. to black-body radiation at the temperature T or

$$\mathcal{J} = B(T),$$

and any excess of radiation energy will be forced over the boundary to the surroundings. We shall have in fact

$$q/k = \{B(T) - B(T')\} / k\rho h.$$

The equation (5.2) yields two equations:

$$\{B(T_1) - B(T_2)\} / k\rho_1 h_1 = \xi_1 \rho \left\{ L_1 \frac{dT_1}{d\tau} + M_1 \frac{dp}{d\tau} + \xi_1 \frac{d\xi_1}{d\tau} \right\}, \quad (5.51)$$

$$\{B(T_1) - B(T_2)\} / k\rho_2 h_2 = -\xi_2 \rho \left\{ L_2 \frac{dT_2}{d\tau} + M_2 \frac{dp}{d\tau} + \xi_2 \frac{d\xi_2}{d\tau} \right\}. \quad (5.52)$$

But since $a_2 h_1 = a_1 h_2$ and $a_1 \rho_1 \xi_1 + a_2 \rho_2 \xi_2 = 0$ we must have

$$L_1 \frac{dT_1}{d\tau} + M_1 \frac{dp}{d\tau} + \xi_1 \frac{d\xi_1}{d\tau} = L_2 \frac{dT_2}{d\tau} + M_2 \frac{dp}{d\tau} + \xi_2 \frac{d\xi_2}{d\tau} \quad (5.53)$$

$$= L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau}, \quad (5.54)$$

equation (5.54) defining the quantity $\xi \frac{d\xi}{d\tau}$. From (5.51) or (5.52) we have

$$4\gamma acT^4 = k\rho \cdot h_1 \xi_1 \rho_1 \left\{ L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau} \right\}. \quad (5.55)$$

We can now reconstruct the condition $\left| \frac{dT}{d\tau} \right| > \left| \frac{dT}{d\tau} \right|_s$ for convection to occur. Since γ and ξ_1 are essentially positive, convection can only occur if

$$\left\{ L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau} \right\}$$

is a positive quantity. Suppose for the moment that $\xi \frac{d\xi}{d\tau}$ is negative or negligible. Then since M is negative, the required condition is

$$\left| \frac{dT}{d\tau} \right| > \left| \frac{M}{L} \frac{dp}{d\tau} \right|.$$

Consider the adiabatic gradient $\left(\frac{dT}{d\tau} \right)_s$. Let Γ be the ratio of the specific heats. Then

$$\left(\frac{dT}{d\tau} \right)_s = \frac{\Gamma - 1}{\Gamma} \frac{T}{p} \frac{dp}{d\tau}.$$

But from (5.41) and (5.42)

$$\left| \frac{M}{L} \right| = \frac{T}{p} \cdot \frac{1 + \frac{1}{2}\gamma(1-\gamma) \left(\frac{\chi}{RT} + \frac{5}{2} \right)}{\frac{5}{2} + \frac{1}{2}\gamma(1-\gamma) \left(\frac{\chi}{RT} + \frac{5}{2} \right)^2} = \frac{T}{p} \cdot \frac{\Gamma - 1}{\Gamma}.$$

(See Unsöld, *loc. cit.*, equation 14, for the expression for Γ .) Hence the condition for convection to occur is

$$\left| \frac{dT}{d\tau} \right| > \left| \frac{dT}{d\tau} \right|_s,$$

but it may be violated if $\xi d\xi/d\tau$ is positive and appreciable. This can only happen, however, if there is a reversal of the density gradient, in the important case of cylindrical flow.

We have now two equations (4.6) and (5.55) for four independent quantities $\frac{dT}{d\tau}$, ξ , ρ and γ . We suppose that a and h are accessible to observation, but in a complete theory of convection they would be determined by boundary conditions. We need two further equations, a pressure equation corresponding to the hydrostatic equation $dp/d\tau = g/k$ and a dynamical equation for ξ . The nature of these will depend on the process by which convection occurs. There have been two main suggestions: Plaskett's theory that convection occurs in a definite cellular structure similar to that observed by Benard in small scale convection phenomena, and Siedentopf's theory that upward moving convection masses push through inert (or slowly downward moving) matter.

6. In Plaskett's theory the flow (except near the boundary) is strictly cylindrical. Then the equation of hydrodynamical continuity reduces to $\xi\rho$ constant, giving

$$\xi_1 \frac{d\rho_1}{d\tau} + \rho_1 \frac{d\xi_1}{d\tau} = 0, \quad (6.11)$$

$$\xi_2 \frac{d\rho_2}{d\tau} + \rho_2 \frac{d\xi_2}{d\tau} = 0. \quad (6.12)$$

The equation of fluid motion for either stream is $\xi \frac{d\xi}{dx} = -g - \frac{1}{\rho} \frac{dp}{dx}$, from which, using $d\tau = -k\rho dx$, we get two equations:

$$\rho_1 \xi_1 \frac{d\xi_1}{d\tau} = \frac{g}{k} \frac{\rho_1}{\rho} - \frac{dp}{d\tau}, \quad (6.21)$$

$$\rho_2 \xi_2 \frac{d\xi_2}{d\tau} = \frac{g}{k} \frac{\rho_2}{\rho} - \frac{dp}{d\tau}. \quad (6.22)$$

Let $\xi_1 \rho_1 = a_2 C$, where C is a constant. Then $\xi_2 \rho_2 = -a_1 C$. Multiply (6.21) by a_1 and (6.22) by a_2 and add. We get

$$a_1 a_2 C \left(\frac{d\xi_1}{d\tau} - \frac{d\xi_2}{d\tau} \right) = g/k - \frac{dp}{d\tau}, \quad (6.3)$$

which gives $dp/d\tau$ when the accelerations are known. If they are negligible (6.3) reduces to the hydrostatic equation $dp/d\tau = g/k$. We have, however, expressions for the accelerations in terms of the density gradient, namely (6.11) and (6.12). Differentiating the equation of state

$$\ln p = \ln \rho + \ln(1 + y) + \ln T + \text{const},$$

we get

$$\frac{1}{p} \frac{dp}{d\tau} \left(1 - \frac{p}{1+y} \left(\frac{\partial y}{\partial p} \right)_T \right) = \frac{1}{\rho} \frac{d\rho}{d\tau} + \frac{1}{T} \frac{dT}{d\tau} \left(1 + \frac{T}{1+y} \left(\frac{\partial y}{\partial T} \right)_p \right),$$

which we may write

$$\frac{N}{p} \frac{dp}{d\tau} = \frac{1}{\rho} \frac{d\rho}{d\tau} + \frac{Q}{T} \frac{dT}{d\tau}.$$

Hence, using (6.11),

$$\frac{1}{\xi_1} \frac{d\xi_1}{d\tau} = \frac{Q_1}{T_1} \frac{dT_1}{d\tau} - \frac{N_1}{p} \frac{dp}{d\tau},$$

with a similar expression for the downward stream. Neglecting small terms (6.3) becomes

$$g/k - \frac{dp}{d\tau} = \frac{a_1 a_2 C^2}{\rho} \left\{ \frac{Q}{T} \frac{dT}{d\tau} - \frac{N}{p} \frac{dp}{d\tau} \right\}. \quad (6.4)$$

The hydrostatic equation $dp/d\tau = g/k$ is a good approximation if $\xi_1 \xi_2$ is small compared with RT ; roughly speaking, if the velocities are substantially smaller than 10 km./sec. We shall use the hydrostatic equation in all subsequent work.

Eliminating $dp/d\tau$ from (6.21) and (6.22) we get

$$\rho_1 \xi_1 \frac{d\xi_1}{d\tau} - \rho_2 \xi_2 \frac{d\xi_2}{d\tau} = \frac{g}{k} \frac{\rho_1 - \rho_2}{\rho}. \quad (6.5)$$

Since the right-hand side of this equation is small, the left-hand side must also be small; either $a_1 = a_2$ so that the left-hand side is a difference between nearly equal quantities or else ξ is only a few cm./sec. (even with γ unity). We shall suppose that $a_1 = a_2$ (in an exhaustive treatment, the equations should include terms in $a_1 - a_2$ to be determined by the boundary conditions).

Using the symbol Δ to denote a difference between the upward and downward streams, equation (6.5) becomes

$$C^2 \left\{ \Delta \left(\frac{Q}{\rho T} \frac{dT}{d\tau} \right) - \frac{1}{p} \frac{dp}{d\tau} \Delta \left(\frac{N}{\rho} \right) \right\} = \frac{4g}{k} \frac{\Delta \rho}{\rho}. \quad (6.51)$$

But (5.53) may be written

$$\Delta \left\{ L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau} \right\} = 0. \quad (6.52)$$

The quantity $\Delta \left(\frac{dT}{d\tau} \right)$ may be eliminated from (6.51) and (6.52). Using the fact that

$\Delta \left(\xi \frac{d\xi}{d\tau} \right) = \frac{dp}{d\tau} \frac{\Delta \rho}{\rho^2}$, the result of the elimination is

$$\frac{Q}{\rho L T} \frac{dT}{d\tau} \Delta \left(\frac{\rho L T}{Q} \right) + \frac{dp}{d\tau} \left\{ \frac{\Delta M}{L} + \frac{\rho T}{p Q} \Delta \left(\frac{N}{\rho} \right) + \frac{1}{\rho L} \frac{\Delta \rho}{\rho} \right\} + \frac{\rho T}{Q} \frac{4g}{k C^2} \frac{\Delta \rho}{\rho} = 0. \quad (6.61)$$

It is to be used in conjunction with the pair

$$\text{from (4.6),} \quad T_0^4 = \frac{8}{3} T^3 \frac{dT}{d\tau} + \frac{C_p T}{2ac} \gamma C, \quad (6.62)$$

from (5.55),

$$8\gamma ac T^4 = k\rho h C \left\{ L \frac{dT}{d\tau} + M \frac{dp}{d\tau} + \xi \frac{d\xi}{d\tau} \right\}. \quad (6.63)$$

These three equations determine C (a constant) and γ and $dT/d\tau$ (functions of τ). The quantity $\xi d\xi/d\tau$ in (6.63) may be taken equal to $-\xi^2/\rho dp/d\tau$ which is calculable in terms of C , namely $-\frac{C^2}{4\rho^3} \frac{dp}{d\tau}$.

Unfortunately it is not possible to obtain a solution for C without an appeal to boundary conditions, and it is clear that the boundary conditions must involve a breakdown of the cylindrical flow, *i.e.* of the condition C constant. A much more elaborate treatment is called for. Nevertheless, the differential equations as they stand can be made to yield some information, as we know that $dT/d\tau$ must lie between certain limits (paragraph 2) and hence also T must lie between certain limits (the radiative (Milne) and adiabatic values). Accordingly a maximum value for the product $C\gamma$ is obtained from (6.62) by inserting the minimum (adiabatic) value for $dT/d\tau$. Further, the ratio C/γ must lie between certain limits which may be computed from (6.63), so that limiting values of C and γ can be determined for any optical depth.

7. According to the ideas of Siedentopf convection does not take place in cells but in irregular bubbles the size of granula. As each bubble rises it dissipates momentum by pushing matter aside, and experiences a resistance. Following Unsöld's account * a bubble of cross-section F moving upwards with a velocity ξ experiences a resistance of order $F\rho\xi^2$. The upthrust is $F\rho g l dp/\rho$, where l is the length of the bubble and $dp/\rho = -dT/T$ (thus ignoring changes in γ). Unsöld equates upthrust to reaction and arrives at the approximate relation

$$\xi \approx \sqrt{g l dT/T}. \quad (7.11)$$

He appeals to observation of the brightness of the granula for dT/T and supposes that l is the thickness of the convection zone which he estimates at 500 km. Then with $dT/T = 2 \cdot 10^{-2}$ he gets $\xi = 1.6$ km./sec. These figures agree with Grottrian and ten Bruggencate's observations of the mean life of a granulum, namely 3.2 minutes, since $l/\xi = 5.2$ minutes, which is the right order of magnitude.

The two models, Plaskett's and Siedentopf's, may be regarded as limiting cases of complete systematisation and complete chaos in the convection currents. The granulation exhibits no regular hexagonal structure and it is observed that the life of a granule is finite.

To return to Siedentopf's model, in our notation $dp/\rho = \gamma Q$ and the hydrodynamical relation corresponding to (6.61) is the simple one

$$\xi_1^2 = g l \gamma Q. \quad (7.12)$$

To get rid of the quantity l we may note that if t is the life of a granulum $l = t\xi_1$ and the equation is

$$\xi_1 = g t \gamma Q \quad (7.13)$$

or, since $\xi_1 \rho_1 = a_2 C$,

$$a_2 C = g t \rho \gamma Q. \quad (7.14)$$

This equation corresponds to equation (6.61) and gives a relation between C and γ which does not, in this case, involve $dT/d\tau$. Equations (6.62) and (6.63) apply to Siedentopf's model if one assumes that the flow is cylindrical. But the simple device of

* *Physik d. Sternatmosphären*, p. 384.

equating upthrust to reaction is too crude to allow us to use (7.14) as it stands in a solution of the differential equation. Here again a much more elaborate discussion is called for.

8. We now deduce limits to the extent of the convection from (6.62) and (6.63). From paragraph 2 we have $T_M \geq T > T_S$ and we can without serious error take $T_M = T = T_0(1 + \frac{3}{2}\tau)^{\frac{1}{2}}$ for moderate values of τ .

We have also

$$\frac{dT}{d\tau} > \left(\frac{dT}{d\tau}\right)_S \quad \text{and} \quad \left(\frac{dT}{d\tau}\right)_S = -\frac{g}{k} \frac{M}{L},$$

hence from (6.62)

$$C\gamma < \frac{2acT_0^4}{C_p T} \left[1 + \frac{g}{k} \cdot \frac{M}{L(dT/d\tau)_M} \right]. \quad (8.1)$$

In terms of $\beta(\tau)$ this equation takes the simple form

$$\beta(\tau) < 1 - \left(\frac{dT}{d\tau}\right)_S \left/ \left(\frac{dT}{d\tau}\right)_M \right. \quad (8.11)$$

Now we have from (6.63) using $\xi \frac{d\xi}{d\tau} = -\frac{C^2}{4\rho^3} \frac{d\rho}{d\tau}$,

$$8\gamma acT^4 = k\rho hC \left\{ L \frac{dT}{d\tau} + M \frac{d\rho}{d\tau} - \frac{C^2}{4\rho^3} \frac{d\rho}{d\tau} \right\}. \quad (8.2)$$

But from (4.2),

$$2acT_0^4 \beta(\tau) = C\gamma \cdot C_p T.$$

Hence

$$\frac{16a^2c^2T_0^4T^3\beta(\tau)}{k\rho hL} - C^2 \left\{ L \frac{dT}{d\tau} + \frac{g}{k} M \right\} + \frac{C^4}{4\rho^3} \frac{d\rho}{d\tau} = 0. \quad (8.21)$$

This equation will only give real roots for C if

$$\left\{ L \frac{dT}{d\tau} + \frac{g}{k} M \right\}^2 > \frac{1}{\rho^3} \frac{d\rho}{d\tau} \cdot \frac{16a^2c^2T_0^4T^3\beta(\tau)}{k\rho hL}. \quad (8.22)$$

This inequality gives an upper limit for β obtained by giving $dT/d\tau$ its maximum value $(dT/d\tau)_M$. Since (8.22) prevents $dT/d\tau$ from approaching anywhere near its adiabatic value $(dT/d\tau)_S$ the inequality (8.22) sets a much lower limit to β than does (8.11).

Table II shows calculations for a pure hydrogen atmosphere in which $g/k = 10^4$ and $h = 9 \cdot 10^7$ cm., corresponding to a diameter of 5 seconds of arc for a granulum.

TABLE II
Maximum Convection for a Pure Hydrogen Atmosphere

τ	4	5	6
T	7856	8247	8589
$L \left(\frac{dT}{d\tau}\right)_M / R$	2088	2165	2258
$\frac{g}{k} \cdot M / R$	-2196	-1935	-1769
$\left\{ L \left(\frac{dT}{d\tau}\right)_M + \frac{g}{k} M \right\} / R$	-ve	+ 230	+ 489
β max. from (8.11)	...	0.1060	0.2167
β max. from (8.22)	...	0.00043	0.00371

The quantity $\left\{ L \left(\frac{dT}{d\tau} \right)_M + \frac{g}{k} M \right\}$ vanishes at $\tau = 4.30$, which is accordingly the top of the convection zone. (It is clearly incorrect to say anything about the bottom of the convection zone without solving a differential equation for T , equations (6.62) (6.63) and (6.61) or (7.13), because the supposition $T = T_0(1 + \frac{3}{2}\tau)^{\frac{1}{2}}$ will be gravely in error at the bottom of the zone. Hence the extent of the convection cannot be accurately estimated.)

Equation (3.81) gives the effect of any prescribed degree of convection on the darkening of the limb (provided that the prescribed convection can be adequately represented by the simple scheme of paragraph 3). It will be shown that the effect on the darkening of the limb of the convection allowed by (8.11) is too small to be detected by present technique, so that the convection allowed by the more stringent (8.22) is *a fortiori* much too small to be detected.

The maximum convection allowed by (8.11) is $\beta = 0$ at $\tau = 4.30$, $\beta = 0.09$ at $\tau = 5$ and $\beta = 0.21$ at $\tau = 6$. Disregarding the depths below $\tau = 6$ this is exceeded by the model (in the notation of paragraph 3) $\tau_1 = 4.30$, $\beta = 0.27$, $\tau_2 = 6$. (This gives $\beta = 0.09$ at $\tau = 5$.)

TABLE III

Effect of Maximum Convection on Solar Limb Darkening

$\cos \theta$	1.0000	0.8352	0.6614	0.4841	0.3122
Radiative Equilibrium	10,000	9011	7968	6905	5873
Maximum Convection (8.11)	10,000	9016	7973	6909	5877

The results are shown in Table III. The distribution of light on the solar disc cannot be distinguished from that due to radiative equilibrium with present technique, all the differences amounting to less than one part in a thousand.

It is possible to demonstrate without reference to cylindrical flow that convection on the scale of paragraph 3 leads to unacceptable velocities. The very simple equation (4.5) gives

$$acT_0^4\beta(\tau) = 2a_1\rho_1\xi_1C_p\gamma T,$$

which gives values of $\xi\gamma$ corresponding to a given β . Now suppose $\beta = 0$ for $\tau < 1.25$ and $\beta = 0.3$ for $\tau > 1.25$. Then $T = 6410^\circ$ at $\tau = 1.5$. With $k = 6.5$ (Unsöld's value; we have hitherto taken $k = 2.74$), $p = 6.3 \cdot 10^{-3}$ atmospheres and $\gamma = 2 \cdot 10^{-3}$ for hydrogen and $\gamma = 0.68$ for a metal of ionisation potential 6.8 volts. The value of C_p is 3.23 R for hydrogen and 4.3 R for the metal (mass 20). Then if there are 60 hydrogen atoms for every metal atom $C_p = 3.25 \text{ R}$ (notice that the result differs very little from the result for pure hydrogen). Further, $\rho \text{ R} = 1.31$. This gives

$$\xi\gamma = 1.38 \cdot 10^6.$$

Thus with $\gamma = 1$, $\xi = 13.8 \text{ km./sec.}$ and with $\gamma = 0.1$, $\xi = 138 \text{ km./sec.}$ The observed contrast of the granulation suggests smaller values of γ , requiring still higher velocities, but even 13.8 km./sec. at $\tau = 1.5$ would completely blur the Fraunhofer lines.

9. APPENDIX. *The darkening of the limb due to an atmosphere consisting of gray material in radiative equilibrium but exhibiting absorption lines.*

Taking Eddington's standard case (η constant) if we assume $B_\nu = a_\nu + b_\nu\tau$ the intensity $\mathfrak{F}_\nu'$ inside an absorption line is given by

$$\mathfrak{F}_\nu' = a_\nu + b_\nu\tau + \frac{2b_\nu - 3a_\nu(1 + \eta)}{3(1 + \eta) + 2q} e^{-q\tau}, \quad (9.1)$$

where $q = 3(1 + \eta)(1 + \epsilon\eta)$ and η, ϵ are positive constants ($\epsilon < 1$). Outside the line

$$\mathfrak{F}_\nu = a_\nu + b_\nu\tau + \frac{2b_\nu - 3a_\nu}{3 + 2\sqrt{3}} e^{-\sqrt{3}\tau}, \quad (9.2)$$

which may be regarded as a special case of (9.1) with $\eta = 0$. The net flux is given by

$$3(1 + \eta)H'_\nu = b_\nu + \frac{3a_\nu(1 + \eta) - 2b_\nu}{3(1 + \eta) + 2q} qe^{-q\tau} \quad (9.3)$$

and, outside the line,

$$3H_\nu = b_\nu + \frac{3a_\nu - 2b_\nu}{3 + 2\sqrt{3}} \sqrt{3} e^{-\sqrt{3} \cdot \tau}. \quad (9.4)$$

Integrating (9.4) with respect to ν ,

$$3H = b + \frac{\sqrt{3} e^{-\sqrt{3} \tau}}{3 + 2\sqrt{3}} \int (3a_\nu - 2b_\nu) d\nu,$$

and since H is constant in radiative equilibrium $H = \frac{1}{3}b$ and $\int (3a_\nu - 2b_\nu) d\nu = 0$. Now the mean absorption coefficient $\bar{k}$ is defined by $\bar{k}H = \int k_\nu H_\nu d\nu$. Since for gray material $kH = k$ outside the lines and is equal to $k(1 + \eta)$ inside a line

$$\bar{k}H = kH + k \int \{(1 + \eta)H'_\nu - H_\nu\} d\nu.$$

Using (9.3) and (9.4) this becomes

$$\bar{k} = k \left[1 + \frac{1}{b} \int \frac{a_\nu q \eta e^{-q\tau}}{1 + \eta + \frac{2}{3}q} d\nu + \frac{1}{b} \int (3a_\nu - 2b_\nu) \left(\frac{qe^{-q\tau}}{3(1 + \eta) + 2q} - \frac{\sqrt{3} e^{-\sqrt{3} \tau}}{3 + 2\sqrt{3}} \right) d\nu \right].$$

The first integral yields a positive quantity. The second integral vanishes if the absorption lines are distributed at random, or are equally spaced. It yields a positive quantity if they are concentrated on the violet side of the limit $2b_\nu > 3a_\nu$ which we take to be the case. We conclude that

$$\bar{k} = k(1 + n),$$

where n is a positive quantity.

When we wish to discuss the darkening of the limb in integrated light we have to treat separately the contributions to the integral coming from frequencies inside and outside the lines. If the integral is physical, that is to say if the darkening of the limb is measured by some instrument which has no dispersion, the contributions from wave-lengths inside the lines will count: but if the integration is mathematical and is obtained by suitably weighting and summing the results of observations made in monochromatic light outside the lines, the contributions from wave-lengths inside the lines will not be counted. The latter is the practical procedure which would be adopted nowadays, but Abbot made no attempt to go "between the lines." We shall discuss the simpler case first—not counting contributions from wave-lengths inside the lines.

For a wave-length between the lines $j_\nu = k_\nu \mathcal{J}_\nu$ and in radiative equilibrium $j_\nu = k_\nu B(\nu, T)$. The efflux in direction θ in the frequency ν is

$$I_\nu(0, \theta) = \int d\tau_\nu \sec \theta B(\nu, T) e^{-\tau_\nu \sec \theta}.$$

As τ is independent of ν if we do not count contributions from the absorption lines, we can carry out a ν integration before making the τ integration and we get

$$I(0, \theta) = \int d\tau \sec \theta B(T) e^{-\tau \sec \theta}.$$

Now $B(T) = H(2 + 3\bar{\tau}) = H(2 + 3\tau(1 + n))$ leading to

$$I(0, \theta) = H(2 + 3(1 + n) \cos \theta).$$

Thus the darkening of the limb in integrated light is increased by the presence of the absorption lines, since n is positive.

Turning to physical integration (counting contributions from between the lines) it is perhaps wiser to appeal to observation than to attempt any computations. According to Miss Adam * weak absorption lines increase their equivalent breadths at the limb (and so increase the darkening of the limb), while strong lines act in the opposite manner, the division between the two classes occurring at about Rowland number 3. On this account there is probably very little difference between "mathematical" and "physical" integrations, and one may suppose that Abbot's observations require very little if any correction to make them refer to the "spectrum between the lines."

In conclusion the writer acknowledges with pleasure the help he has received from Mr. D. W. N. Stibbs, who has checked the arguments and calculations in the entire paper.

Summary

This paper is mainly concerned with the effect of convection on the darkening of the limb of the Sun. It also gives limits to the amount of convection possible. In paragraph 3 an empirical analysis is made of the amount of convection required to bring about agreement between theory and Abbot's observations of limb darkening (integrated by Milne) assuming the solar atmosphere to be gray. Paragraphs 4 to 7 sketch a theory of convection from which it is possible to deduce maximum values for convection and corresponding minimum velocities for the convection streams. In paragraph 8 the effect of the maximum convection on the darkening of the limb is found to be less than one part in a thousand, and the empirical convection of paragraph 3 is found to require velocities which would blur the Fraunhofer lines. An appendix (paragraph 9) discusses the effect of the Fraunhofer lines on the darkening of the limb.

*Commonwealth Solar Observatory,
Canberra :
1941 January.*

* *M.N.*, 98, 112, 1938, and 100, 595, 1940.

THE BIRTH DISTRIBUTION OF SUNSPOTS ON THE SUN'S DISC

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1. The observation that the birthplaces of sunspots are asymmetrically distributed with respect to the Sun's central meridian has been the subject of frequent discussions up to the present time. Some authors suggested that the higher frequency of first appearances of sunspots on the eastern half of the Sun was due to some influence exercised by the Earth. A. Schuster showed, however, in an important paper * that the eastern excess could be explained as an apparent effect. He assumed that owing to foreshortening the true size of the smallest detectable sunspots must increase from the centre of the Sun's disc towards the limb. The length of time which elapses between the real birth of a sunspot and its first visibility is therefore a function of the longitude from the central meridian, and the apparent birth rate is different from the true birth rate. In this way Schuster could derive an eastern excess of first appearances from an even distribution of real births.

W. Gleissberg † and M. Waldmeier ‡ developed theories to determine the time lag between real birth and first appearances. They found values of 0.3 to 3 days for spots at the centre of the disc. However, as J. C. Dobbie § pointed out, such high values are inconsistent with the initial rate of growth as derived by him from an analysis of Greenwich sunspot measures.

The present writer has found good reason to believe that the theories of Gleissberg and Waldmeier are not sufficient to derive the above time lag from the observed eastern excess of first appearances. Gleissberg assumes that the minimum size of a detectable sunspot varies with the secant of the distance from the centre of the Sun's disc. As the present author has shown ||, there exists besides the geometrical foreshortening a very remarkable physical foreshortening which affects the visibility of a spot to a high degree. The simple secant law has to be abandoned. Moreover, the assumption of a constant rate of growth of all spots seems to be unjustified. In the following discussion we shall try to represent the frequency curve of observed first appearances by a theory based on more general assumptions than hitherto adopted.

2. A slight modification of a graphical method described by M. Minnaert ¶ has been found to be most suitable for our purpose. Minnaert's construction gives, for any individual form of life-curves of sunspots, the number of spots which reach the minimum size necessary to become detectable in a given interval of longitude. This theoretical number can only be compared with the one observed at that longitude if the first appearances of sunspots were recorded by continuous observation. Since most available statistics are based on daily observations, the first appearances are registered too late, that is in a more westerly position. Spots having a duration of less than 24 hours may escape observation altogether.

The following procedure has been adopted to make the theoretical results comparable with the observations. Since the mean daily synodic motion of a sunspot is $13^{\circ}.2$, a spot will be seen, in the case of daily observations, for the first time somewhere in the interval of $13^{\circ}.2$ following the point where it reaches the limit of visibility. We assume that the probability of seeing the spot is the same in every point of this interval,

* *Proc. Roy. Soc., A*, 85, 309, 1911.

† *Astr. Mitt. Zürich*, No. 138, 460, 1939.

|| *M.N.*, 100, 645, 1940.

‡ *A.N.*, 263, 13, 1937; 268, 81, 1939.

§ *Observatory*, 62, 289, 1939.

¶ *A.N.*, 269, 48, 1939.

provided, of course, that the spot remains above the curve of visibility throughout the interval.

As an example we treat the case of a spot which reaches the stage of visibility at an eastern longitude of 72° from central meridian. If its duration is longer than 24 hours, the probability of being first seen in the longitude zones -80° to -70° , -70° to -60° and -60° to -50° are $2/13.2$, $10/13.2$ and $1.2/13.2$ respectively. If the duration of visibility is shorter than 24 hours, only that interval of longitude has to be considered in which the spot remains above the curve of visibility. If, for instance, this interval ends at longitude -66° , the probabilities of registering the first appearance of the spot in the above-mentioned zones would be $2/13.2$, $4/13.2$ and 0 respectively. Since the total probability of observing the spot is only $6/13.2$, the chances are in favour of such a spot being missed altogether.

3. The true or "normal" size which a spot must reach to become visible at a given longitude from central meridian has been defined as "curve of visibility." This curve is, of course, dependent upon the method of observation. Since in the present investigation the results will be compared with Greenwich observations, the curve of visibility derived for the Greenwich instruments has been applied.*

4. For facilitating the calculation of the theoretical birth distribution, the different forms of individual life-curves of sunspots were classified according to length of life. Since the visibility of a group is determined not by the total area, but by the area of the largest spot in the group, the latter area has been chosen for deriving life-curves of sunspot groups from the material published in the *Greenwich Photoheliographic Results*.

Although the scatter of the individual life-curves in a group of given length of life is very great, the following general rules were found from a large amount of material:—

- (1) The life-curves are asymmetrical, the period of growing being shorter than that of decline.
- (2) The initial rate of growth increases with increasing lifetime.
- (3) The rate of decline is about the same for spots of different lifetimes.
- (4) The average maximum of the life-curves increases steadily (nearly linearly) with increasing lifetime.

These results are similar to those derived by W. Moss for the life-curves of individual spots. I am indebted to Mr. Moss for showing me his results before publication.

Life-curves of spots lasting longer than 30 days have been taken from the Greenwich publication in *M.N.*, 85, 553, 1925. In this paper diagrams which show the characteristic form of life-curves have been published. In accordance with the laws stated above, life-curves of spots, not derivable from direct observation, were constructed for life durations of $1/2$ day and 120 days as well as for spots of 12 days to 23 days, whose birth and death occurs nearly or entirely on the Sun's invisible hemisphere. The maximum height of an average life-curve of 3 days duration is 15 millionths of the Sun's hemisphere, that of a life-curve of 9 days duration is 50 millionths. The ratio of the rising part of the life-curve of 9 days duration to the declining part has been found to be approximately 4 to 7.

5. Each form of life-curve yields a characteristic birth distribution which will contribute to the observable distribution according to the number of groups belonging to this class. It is therefore necessary to estimate the number of groups having a given length of life. For this purpose use has been made of Mrs. Maunder's Table V †, from which the number of groups born in the central lunes has been derived. It is found that the number of spots decreases rapidly with increasing lifetime. More than half of all groups have a length of life less than 2 days, and 90 per cent. less than 11 days. The following selection of life-curves was considered to represent the real distribution near enough. The number of groups is set in brackets behind the length of life. In

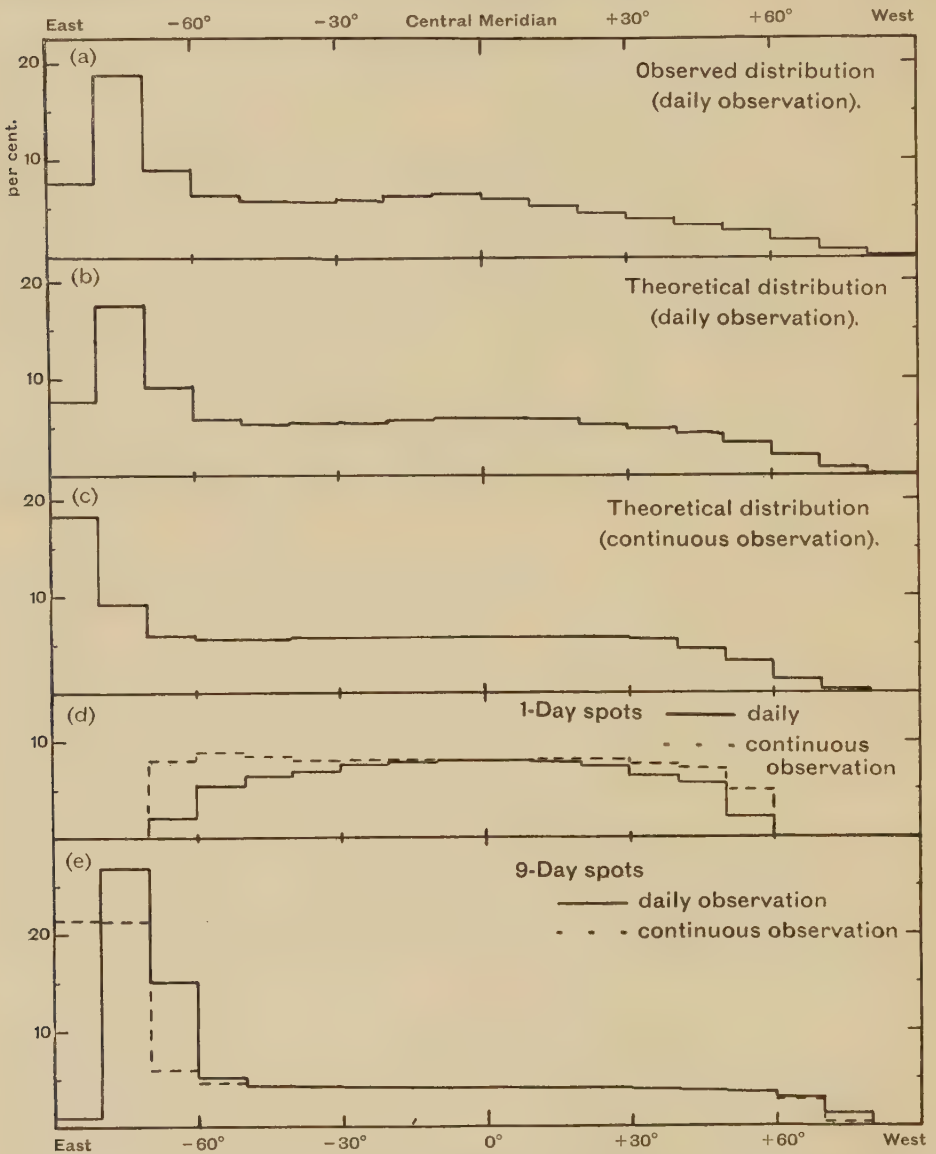
* *M.N.*, 100, 650, 1940.

† *M.N.*, 67, 456, 1907.

comparing these numbers it must be kept in mind that the steps between the chosen lifetimes are increasing

1/2 day (90), 1 day (50), 2 days (29), 3 days (20), 5 days (37), 7 days (14), 9 days (12), 12 days (7), 15 days (4), 18 days (5), 23 days (5), 30 days (2), 45 days (2), 60 days (1), 120 days (1/2).

6. In the accompanying figure, curve (a) represents the birth distribution in longitude of the sunspot groups observed at Greenwich in the years 1886 to 1935. The first



The birth distribution of sunspots.

appearances of all groups regardless of their duration have been included. Recurrent groups have been listed in each appearance. For some years use could be made of counts published by Akesson * and by Waldmeier.† Akesson's figures had to be supplemented

* *Lund Medd.*, Serie II, No. 11, 1914.

† *Astr. Mitt. Zürich*, No. 138, 460, 1939.

by the second and following appearances of recurrent groups and Waldmeier's figures by those of 1-day groups. The observed frequency distribution shows a high maximum in the second zone, that is in the eastern longitudes of 80° to 70° . Nearly 19 per cent. of all first observations fall into this interval. Near the central meridian appears a small secondary hump. Since the curve is based on more than 12,000 observations, the mean error of a single point is so small that this central hump may be regarded as real. Its height amounts to 6 ± 0.28 per cent. Curve (b) shows the theoretical distribution of births in the case of daily observations as calculated by the author's method. Curve (c) gives the distribution as it would be observed if continuous observations of the Sun were carried out. As can be seen from the figure, the two first curves are in good agreement with each other. We conclude therefore that the present theory is sufficient to explain the observed frequency distribution.

In the figure are included as curves (d) and (e) the frequency distributions of average 1-day and 9-day groups, both for the case of daily and continuous observation. These examples show that the small hump of the observed curve near the central meridian is due to the small spots of short real lifetime. The maximum near the east limb is produced by the larger spots of long length of life.

In conclusion the author wishes to acknowledge his sincerest thanks to Professor F. J. M. Stratton for the suggestion of this paper.

Summary

The distribution of the first appearances of sunspots as a function of the longitude from the central meridian is dependent on the curve of visibility, on the form and number of the life-curves of spots, and on the interval in which the observations of the Sun are made. By making reasonable assumptions as regards these three factors, the observed distribution has been represented theoretically by the use of a method described in the paper.

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A REMARKABLE SPOT ON JUPITER

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During the apparition of 1940-41 there has been under observation at the north edge of Jupiter's S. Temperate Belt a dusky spot, which has displayed a motion so remarkable as to merit special comment.

For about twenty days after it was first recorded on July 31 it exhibited a rapid motion relative to System II, in the sense of decreasing longitude. Then a "deceleration" set in and it remained almost stationary in System II from August 22 until September 23, after which its longitude began again to decrease at almost its original speed. During the latter part of October and the beginning of November there was once more a tendency to slow down; but by the beginning of December, though it had faded and was becoming rather a difficult object to observe, it was manifestly running away again for the third time.

In all I made twenty-five determinations of the longitude of this spot between July 31 and December 21, after which it had become so faint that I could no longer be certain of its identity. When all these longitudes had been plotted, they suggested very strongly a damped oscillation of some kind, superposed upon a fairly uniform decrease of longitude. Probably this is as much as can legitimately be concluded from the nature of the observations; but rather to beguile the "black-out" evenings than in the hope of arriving at a conclusion of any scientific value, I thought I would try to fit to them the best curve I could obtain that represented strictly a damped harmonic vibration. The success achieved is so striking that I submit herewith the results for what they are worth.

The observations were first given weights, ranging from 1 to 3, these being assigned by reference to the conditions as recorded in the observing book. The period was evidently close to 62 days; and as the mean motion was not quite uniform, it was necessary to introduce a small term in t^2 , when, after some manipulation of the damping coefficient and the phase, the following expression emerged, representing the System II longitude at time t , where t is measured in days and the epoch is 1940 September 8.

$$\lambda = 270^\circ.8 - 0^\circ.274t - 0^\circ.0015t^2 + 4^\circ.77e^{-0.015t} \sin 5^\circ.80t. \quad (1)$$

Table I gives the date, the number of days elapsed from the epoch, the observed longitude, the longitude calculated from the above formula, the residuals in the sense $O - C$, and the assigned weights.

The longitudes were derived from visual estimates of the times of transit of the spot across the central meridian of the planet, recorded to the nearest minute of time. As the planet rotates through $0^\circ.6$ in one minute, they are given to the nearest degree only. I should estimate the probable error of a satisfactory observation to be about two minutes of time or 1.2 degrees of longitude. It will be seen, however, that only three of the residuals exceed this figure and that one only is just twice as great. One observation, recorded on October 17, has not been included in the solution; it was made during an air-raid under conditions of atrocious seeing and is clearly, from the record, of very little value. The residual derived from it is $-4^\circ.9$. There was also a tentative record on December 12; but the observation was rejected as soon as it was made, since the conditions were so poor that the presence of the spot was only suspected.

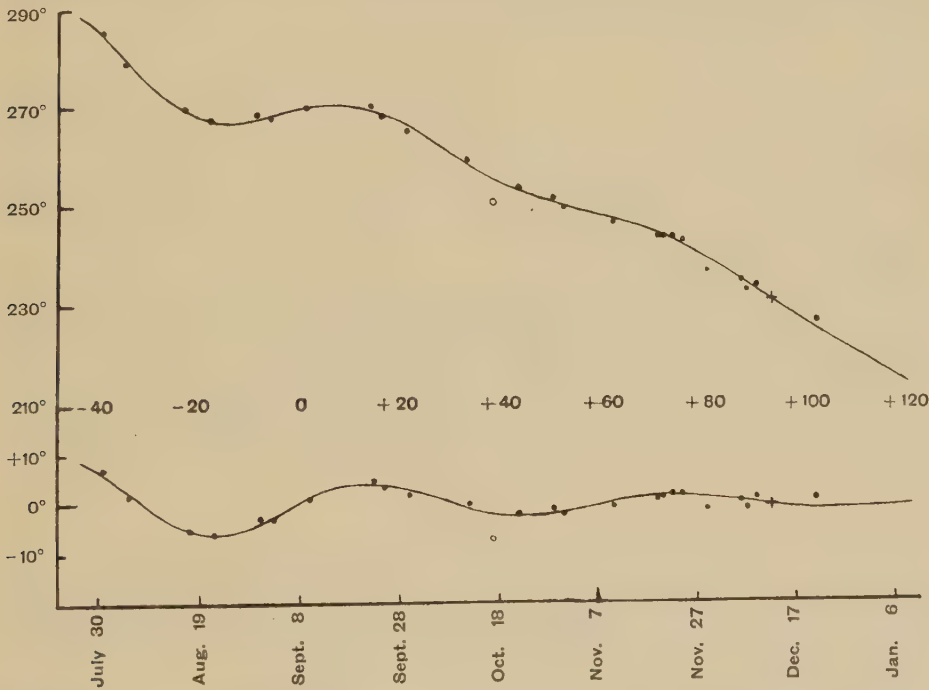
The upper portion of the accompanying figure shows the observed longitudes

TABLE I

Date	<i>t</i>	λ_0	λ_c	O - C	W	Date	<i>t</i>	λ_0	λ_c	O - C	W
1940						1940					
July 31	-39	286°	285°·4	+0°·6	3	Oct. 29	51	252°	250°·9	+1°·1	1
Aug. 5	34	280	280°·7	-0°·7	1	31	53	250	250°·3	-0°·3	3
17	22	271	270°·9	+0°·1	2	Nov. 10	63	247	247°·7	-0°·7	3
22	17	269	268°·9	+0°·1	3	19	72	244	244°·6	-0°·6	1
31	8	270	269°·0	+1°·0	3	20	73	244	244°·2	-0°·2	3
Sept. 3	-5	269	269°·6	-0°·6	3	22	75	244	243°·3	+0°·7	2
10	+2	271	271°·3	-0°·3	3	24	77	243	242°·3	+0°·7	3
23	15	271	270°·1	+0°·9	1	29	82	237	239°·4	-2°·4	2
25	17	269	269°·4	-0°·4	2	Dec. 6	89	235	235°·0	0°·0	3
30	22	266	266°·8	-0°·8	3	7	90	233	234°·3	-1°·3	3
Oct. 12	34	260	258°·9	+1°·1	3	9	92	234	233°·0	+1°·0	3
22	44	254	253°·6	+0°·4	2	21	+104	227	225°·1	+1°·9	3

plotted against the time, the continuous line representing the motion calculated from the formula; in the lower portion the curve represents the oscillatory term only and the observed longitudes have been corrected by subtracting from them the first three terms of the formula, this having the effect of reducing the purely translatory motion of the spot to zero. In both diagrams the solitary rejected observation is shown by a hollow circle. The suspected position on December 12 is indicated by a cross, as it happens to fall almost exactly on the curve.

A few observations have been sent to me by two other observers, Mr. F. J. Hargreaves, of Kingswood, Surrey, and Mr. H. M. Johnson, of Des Moines, Iowa; but they have



not been included in the solution for the following reasons. There is always a certain amount of systematic difference between the transit times of different observers, which is not easy to estimate from only a few observations and which may vary from apparition to apparition; the solution I have given, being based on the observations of one observer alone, is free from this defect. Mr. Hargreaves was unable to observe the region until

after I had lost the spot altogether; as he recorded several minute spots at the north edge of the S. Temperate Belt which I had not seen, it is doubtful whether his first observation refers to the same object, though his second falls exceedingly close to the curve. Mr. Johnson secured seven observations during August and September, but it is unfortunate that just at this time he was endeavouring to correct a tendency, of which he had become aware, to record his transits too early. During such a period of transition the material obtained is hardly likely to be homogeneous. I give these other nine observations in Table II with their residuals, but have not plotted them on

TABLE II

Date	t	λ_0	λ_c	O - C	Observer.	Date	t	λ_0	λ_c	O - C	Observer.
1940-41						1940-41					
Aug. 4	-35	280°	281.7	-1.7	J	Aug. 29	-10	271°	268.7	+2.3	J
14	25	269	272.7	-3.7	J	Sept. 29	+21	270	267.4	+2.6	J
19	20	266	269.9	-3.9	J	Dec. 28	111	229	221.1	+7.9	H
21	18	267	269.2	-2.2	J	Jan. 9	+123	215	214.2	+0.8	H
23	16	270	268.8	+1.2	J						

the diagrams, since the curves, having been computed for my own observations alone, would naturally tend to reduce my residuals at the expense of the others.

It may be asked what possible physical significance can be attached to a result such as has been obtained. The answer is very little, if the harmonic motion found is the result of a mere *tour de force*, designed to wrest from the observations data which they are inadequate to divulge. But if the damped vibration is real, it may possibly provide a clue to some of the fundamental properties of Jupiter's atmosphere.

It has been demonstrated by R. Wildt * and I believe also by myself † that, unless the internal temperature is very high, the density of Jupiter's atmosphere must increase very rapidly with the depth; and it now seems to be generally accepted that in the lower strata the gas of which it is composed is so dense as to be almost indistinguishable from a liquid, save for the absence of a bounding surface. For some time I have thought it not improbable that some, at least, of the surface phenomena may be due to causes deep-seated within the atmosphere and may even be connected with masses of light solids, floating in the denser gases.‡

Imagine such a solid mass, displaced for some reason from the level at which it would float in equilibrium, which has become free to adjust itself. It will oscillate with a strongly damped vibration, which we may assume to be approximately harmonic, about the equilibrium level. But change in level will alter the radius of its rotation about the axis of the planet; and since angular momentum must be conserved, we have $\omega r^2 \cos^2 \phi$ is constant, ω being the angular velocity of rotation, r the distance of the body from the centre of the planet and ϕ the latitude, in this case about -26° ; whence

$$d\omega = -\frac{2\omega}{r} dr. \quad (2)$$

Since ω and r at no time differ sensibly from the angular velocity and radius of Jupiter, $d\omega$ may be considered as proportional to $-dr$. The value of ω therefore will also fluctuate with a damped harmonic vibration.

If ω_0 be the angular velocity of a standard zero meridian and λ the longitude of an object referred to it, $\frac{d\lambda}{dt} = \omega_0 - \omega$; so that in the case considered the gradient of λ is a

* *Veröffentlichungen der Universitäts-Sternwarte zu Göttingen*, Nr. 40, 1934.

† *M.N.*, 97, 574, 1937.

‡ B.A.A. Presidential Address, *B.A.A.J.*, 50, 2, 1939.

damped harmonic vibration. Now the gradient of a damped harmonic oscillation is readily shown to be another damped harmonic oscillation; as a consequence, the object itself will exhibit an oscillation of this nature in longitude.

It is just such a motion in longitude that has been observed on the surface, where it is not impossible that the dusky spot may have represented the effect upon the upper strata of the presence of a solid mass floating at a considerable depth below.

It is of interest to examine a little further the demands which this hypothesis makes upon the physical conditions in the atmosphere. If we neglect the small secular change implied by the term in t^2 , it is easy, by differentiating (1), combining with (2) and inserting the appropriate values of the constants, to determine the difference between the greatest and least depths attained by the hypothetical floating body during the time covered by the observations. This is found to be 52 km., the maximum depth corresponding to $t = -34$ days and the minimum to $t = -3$ days. It will be noted that the vertical oscillation is not exactly 90° out of phase with the horizontal motion, owing to the effect of the damping. By a similar process it may be shown that the maximum vertical velocity attained was only 3.0 cm. sec.⁻¹ upwards at $t = -20$ days.

That an object should continue to oscillate in a strongly resisting medium for four months, with such a comparatively large amplitude and so small a velocity, need not necessarily occasion any misgivings; for the diameter of the spot as observed at the surface was for some of the time at least as large as that of a satellite, say 3000 km., so that the mass and therefore the inertia of the solid body may well have been enormous and, depending on the cube of its linear dimensions, may have easily combated the resistance, which would have been proportional to the square.

At fairly high laboratory pressures the compressibilities of most solids are found to be between 10 and 100 times as small as those of typical fluids. On the present hypothesis, if we neglect the compressibility of the solid, we can obtain an estimate of the density gradient, $\frac{d\rho}{dz}$, and the compressibility, $-\frac{1}{v} \frac{dv}{dp}$, of the fluid, assumed to be constant over the range of depths concerned. For if v and ρ_0 be the volume and density of the solid, ρ the density of the fluid, z the vertical distance in cm. from the equilibrium level and g_J ($=2600$ cm. sec.⁻²) the value of gravity on Jupiter, the force tending to restore the level is $vg_J(\rho_0 - \rho)$ and the acceleration is of the form $\frac{g_J(\rho_0 - \rho)}{\rho_0 + c\rho}$, where the value of c depends on the shape of the body. For a sphere $c = \frac{1}{2}$; for an oblate spheroid with its minor axis vertical c increases with the oblateness. As we have absolutely no knowledge of the form of the floating mass, we may let $c = 1$, its actual value if the solid be a circular cylinder with its axis horizontal; and since ρ and ρ_0 are very nearly equal, we shall simplify by writing $c\rho = \rho_0$. The expression for the acceleration then becomes

$$\frac{g_J(\rho_0 - \rho)}{2\rho_0}.$$

Equating this to the acceleration derived from the period, we have

$$\frac{1300(\rho_0 - \rho)}{\rho_0} = -\frac{4\pi^2 z}{T^2 \sin^2 \alpha},$$

where T is equal to 5.4×10^6 , the period measured in seconds, and α is the angle of the associated logarithmic spiral; whence

$$\frac{d\rho}{dz} = 1.1 \times 10^{-15} \rho_0. \quad (3)$$

Combining (3) with the equation of hydrostatic equilibrium and neglecting the minute difference between ρ and ρ_0 , we obtain for the compressibility

$$-\frac{1}{v} \frac{dv}{dp} = \frac{1}{\rho_0} \frac{d\rho}{dp} = -\frac{1}{\rho_0^2 g_J} \cdot \frac{d\rho}{dz} = 4 \times 10^{-19} \rho_0^{-1}, \quad (4)$$

where p is in dyne cm^{-2} .

There is no way of deducing the value of ρ_0 , but the dynamical researches of Jeffreys and Wildt imply that we cannot be seriously out if we take it to be 0.3. This gives 3.2×10^{-16} gm. cm^{-4} for the density gradient and 1.3×10^{-12} per atmosphere for the compressibility.

The last figure seems to be of altogether the wrong order of magnitude. For pressures up to about 15,000 atmospheres the work of Bridgman shows that the compressibilities of typical fluids tend towards nearly constant limits of the order of 10^{-5} per atmosphere, which is best interpreted on the assumption that the atoms or molecules are themselves compressible, after all the space between them has been squeezed out.* Experiment has not yet informed us of the trend of the compressibilities under considerably higher pressures, such as may well be involved at no very great depth below the surface of Jupiter.

If the compressibility of the solid approaches that of the fluid, the above calculation is of course meaningless; the alkali metals, for example, form an exceptional group of solids with compressibilities tending to stationary values not differing greatly from 10^{-5} . If the compressibility of the solid should exceed that of the fluid, the equilibrium level would be unstable and a body placed there would go either straight up or straight down.

Although we have obtained little from the latter part of the discussion except a strong hint that the hypothesis may be barren, it is well perhaps to place it on record, because of the possibility that the method outlined may be of use in the event of other similar periodicities being observed on Jupiter, or when more is known about the values of compressibilities under exceedingly high pressures.

It might be expected that in the circumstances suggested above the intensity of the spot at the surface would have undergone some fluctuations in sympathy with the changing depth of the floating mass. At the time when the observations were made the determinations of longitude were just a part of the ordinary routine work and there was nothing to suggest that this spot should be singled out for a detailed study of intensity variations. Such notes as were made are therefore only casual. But it is interesting, and may be significant, that on both September 3 and 10, when the definition was "horrible" and "poor" respectively, the spot was recorded as "conspicuous." The date when the floating body would have been nearest to the surface was September 5. On August 31 it had been described as merely "grey."

The next time it came up, to a considerably lower level, of course, the surface spot was very much fainter and there was evidently no sign of intensification; for on November 8 it was missed altogether in very bad definition and on November 10 was recorded as being small and difficult and seen only during a temporary improvement of poor seeing. Thereafter there are sundry references to its being difficult, but by this time there is no doubt that a general fading had set in.

This record of an oscillating motion in longitude appears to be almost unique. It is possible, however, that something similar occurred in latitude about -8° during the apparition of 1928-29. On the chart of the S. Equatorial Spots for that apparition, published at the end of the twenty-seventh Report of the Jupiter Section of The British Astronomical Association, it will be seen that spots Nos. 29 and 30 probably represent one and the same object, as may also Nos. 37 and 38. In both these instances there is a

* P. W. Bridgman, *Physics of High Pressure*, 1931. The laboratory compressibilities were determined at temperatures in the neighbourhood of 60°C .

strong suggestion of a motion similar to that which has been discussed and in the latter case a period of the order of 65 or 70 days seems to be indicated. It is probable that the history of both these objects is worthy of further study.

Summary

The longitudes of a dusky spot at the N. edge of Jupiter's S. Temperate Belt, derived from twenty-five observations of its passage across the central meridian, are plotted against the time. It is found that a damped harmonic oscillation in longitude is very closely represented by the observations.

The suggestion is made that the spot observed at the surface was associated with the presence of a solid body, deep down in the fluid atmosphere, which was performing a vertical oscillation about the level at which it would have floated in equilibrium.

On this hypothesis the density gradient and the compressibility of the atmosphere at the equilibrium level are calculated; but the figure derived for the latter is smaller by a factor of 10^7 than the lowest values that have yet been found for fluids in the laboratory.

Erratum

M.N., 97, 575, 1937.

$$\text{Equation (3). For } \frac{T}{T_0} = \left(\frac{\rho}{\rho_0}\right)^{1-\frac{1}{\gamma}} \text{ read } \frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{1-\frac{1}{\gamma}}.$$

ON THE HEIGHTS AND VELOCITIES OF METEORS

H. C. Plummer

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1. The object of this paper is to discuss the seasonal variation in the heights and frequency of meteors, and the distribution of meteoric velocities. But in relation to the heights it is first necessary to decide upon an appropriate mean, since it has lately been claimed that the arithmetic mean is inferior to the harmonic for this purpose.

The recent preference given to the harmonic mean is based on the belief that the reciprocals of the heights determined are distributed normally. Let $z = u^{-1}$ be the height of a meteor at, for example, the time of appearance. Then if Udu is the frequency between u and $u + du$, and the distribution in u is normal,

$$U \propto \exp. [-h^2(u - u_0)^2]. \quad (1)$$

Hence if Zdz is the frequency between z and $z + dz$, the distribution in z is defined by

$$Z \propto z^{-2} \exp. [-h^2(z^{-1} - H^{-1})^2], \quad (2)$$

where $Hu_0 = 1$. The corresponding frequency curve is roughly indicated in fig. 1.

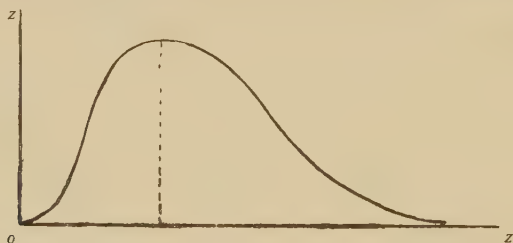


FIG. 1.

Since

$$\frac{1}{Z} \frac{dZ}{dz} = -\frac{2}{z} + \frac{2h^2}{z^2} \left(\frac{1}{z} - \frac{1}{H} \right)$$

the position of the mode is given by

$$Hz^2 + h^2(z - H) = 0. \quad (3)$$

Whatever the relative values of h and H , the positive root of (3) is clearly smaller than either.

2. Here we come to a question of fact. Is the frequency of the heights determined of the type represented by (2)? At a time when the distribution was simply a matter of interest, and not of controversy, I discussed * two samples of Denning's material for the heights of appearance and concluded that the dispersion was in the main normal. Recently McIntosh †, using a larger quantity of Denning's material, has shown graphically that the distribution is symmetrical alike for the terminal and for the intermediate points; for the disappearances alone a disturbance in the neighbourhood of 60 km. is apparent, but this interesting feature is irrelevant to the present question. Other evidence quoted by McIntosh from the results of the Harvard Expedition and of the American Meteor Society reveal skewness in the frequency curve, but in contradictory

* *M.N.*, 76, 392, 1916.

† *M.N.*, 100, 514, 1940.

senses. Thus it cannot be held that any serious departure from symmetry has in fact been established.

3. The next point to be noticed is that use of the harmonic mean has been followed by what is surely a quite unjustified application of it.

Let $z_1, z_2, \dots, z_n$ be a set of heights determined. The difference between the two means is

$$\text{A.M.} - \text{H.M.} = \frac{\sum z_i^{-1} z_j^{-1} (z_i - z_j)^2}{n \sum z_i^{-1}} \geq 0, \quad (4)$$

which is necessarily positive, as is well known, unless there is no dispersion. Hence its magnitude can be regarded as a rough measure of the dispersion. But, contrary to what has been asserted, it has little or nothing to do with the errors of observation. Let ζ be the true height determined with an error ϵ , so that $z = \zeta + \epsilon$. Clearly the dispersion in z arises jointly from the dispersions in ζ and ϵ , and it is doubtful if in any case the latter have been separately determined with any great accuracy. From the two samples of Denning's results already quoted I found the standard deviation of z (appearances) to be just over 12 miles. There is no reason to doubt that this is due mainly to the variation of ζ arising from physical causes, and it is almost certain that the contribution of the error ϵ is small in comparison. Hence to regard the variance of z , or any single index like the difference between the arithmetic and the harmonic mean, as ground for condemning an observer's results is in general without justification. Even if the observations were free from error this difference would still exist.

4. To return to the relative merits of the two means, apart from a wrong interpretation of the difference between them, the point at issue is simple. Since $z = \zeta + \epsilon$ the ordinary mean values are connected by the relation $\bar{z} = \bar{\zeta} + \bar{\epsilon}$. There is no sufficient reason to doubt that the distribution of z , for good determinations, is symmetrical. That has been seen to be true, but it is strictly irrelevant. The real question concerns the distribution of ϵ alone. If it is symmetrical $\bar{\epsilon} = 0$ and $\bar{z} = \bar{\zeta}$. Then all is well, for, whatever the distribution of ζ may be, its mean value has significant interest.

Of course the harmonic mean can be regarded merely as an ordinary weighted mean, the weight of z_i being z_i^{-1} . In other words, the lower heights are given preponderating weight, as McIntosh has pointed out. This system of weights might be appropriate to the errors of determination, though it must be proved in every case, and the proof would be extremely difficult. But it is not specially appropriate to the actual heights with which the errors are entangled. This seems to put the fallacy of the harmonic mean in the clearest light.

The dispersion in height may be relatively large and the samples of data small. In such cases there is one statistic which deserves more consideration than it generally receives, the median. It has a simple meaning and it is easily found. It has the advantage that it is the same whether it is found from the variate itself or from any monotone function of the variate. Thus here, whether the height is chosen or the reciprocal of the height, the height resulting from the median is identically the same. To know even roughly the height which a class of meteors reaches or does not reach in an equal number of cases is to know something about the class. As a last resort in a case of difficulty, the median can be invaluable; not that the work of good visual observers of meteors has in fact presented such special difficulties.

5. Thus in what follows monthly mean heights of non-shower meteors have been taken from Table V* in McIntosh's paper. The result of harmonic analysis is to represent them in the form

$$H = h_0 + \sum h_r \sin(rt + a_r), \quad (5)$$

* *M.N.*, 100, 522, 1940.

where $t/360^\circ$ is the fraction of the year from January 15 and $r = 1, 2, 3, 4$. The figures are:

TABLE I

	h_0 mi.	h_1 mi.	a_1 °	h_2 mi.	a_2 °	h_3 mi.	a_3 °	h_4 mi.	a_4 °
<i>Ha</i> (I):	67.0	2.1	199	1.0	274	1.1	184	0.3	32
(II):	67.3	2.1	184	0.6	323	1.4	181	0.9	27
<i>Hm</i> (I):	55.3	2.6	249	1.3	249	1.0	194	0.5	80
(II):	55.2	2.9	225	1.1	263	1.1	162	1.4	70
<i>Hd</i> (I):	43.7	4.7	264	1.3	254	1.6	199	0.8	45
(II):	43.6	5.2	241	2.1	254	1.4	181	1.8	68

It is to be recalled that *Ha* refers to appearance, *Hd* to disappearance and *Hm* to the mid-point. Group I is based on 820 meteors computed by Denning; in Group II the results of other British computers for 607 meteors have been added to the former. A still wider group, III, has not been used. The work was begun with no particular expectation beyond finding the first harmonics and substituting a periodic form for a graphical representation. But the higher terms seem to deserve inspection, partly because the coefficients remain sensible and partly on account of a certain degree of regularity in the phases.

6. These mean heights (h_0) are not identical with those found by McIntosh:

	(<i>Ha</i>) I	II	(<i>Hm</i>) I	II	(<i>Hd</i>) I	II
h_0	67.0	67.3	55.3	55.2	43.7	43.6
McIntosh	67.7	68.3	56.0	56.3	44.9	45.6

and the reason is simple. In the first line (h_0) the months have equal weight. In the second each meteor is weighted equally, so that the richer months have greater weight, and these months are marked by greater height. Similarly, in my former paper based on two samples of Denning's data 68.0 and 69.8 miles were found for the mean height of appearance. But the samples contained shower meteors which tend to appear at greater heights.

For the main annual term the amplitudes and the rough dates of the corresponding maxima run thus:

	<i>Ha</i>	<i>Hm</i>	<i>Hd</i>
I	2.1 mi., Sept. 26	2.6 mi., Aug. 6	4.7 mi., July 21
II	2.1 Oct. 11	2.9 Aug. 31	5.2 Aug. 14

The two features noticed by McIntosh are very apparent. First, there is the progressive increase in the amplitude with the depth, from 2 to 5 miles. Secondly, the maximum phase falls later in the year for the phenomenon associated with the greater height, the interval being about 2 months between the dates when meteors disappear or appear at the highest average level.

For the six-monthly term the figures are:

	<i>Ha</i>	<i>Hm</i>	<i>Hd</i>
I	1.0 mi., Apr., Oct. 13	1.3 mi., Apr. } 25	1.3 mi., Apr. } 23
II	0.6 Mar., Sept. 19	1.1 Oct. } 19	2.1 Oct. } 23

With a small amplitude in the upper level it is not surprising that one pair of dates is slightly out of line with the rest. It is more surprising that the other five all agree within a fortnight, though not of course a resemblance to be stressed by itself.

The four-monthly term shows this agreement in phase even better and the amplitudes are also more consistent. The figures are:

	<i>Ha</i>	<i>Hm</i>	<i>Hd</i>
I	1.1 mi., Apr. } 14	1.0 mi., Apr. } 10	1.6 mi., Apr. } 9
II	1.4 Aug. } 15 Dec. }	1.1 Aug. } 21 Dec. }	1.4 Aug. } 15 Dec. }

Can it be worth while looking at another term? For this three-monthly period the amplitudes and dates of maxima are:

	<i>Ha</i>	<i>Hm</i>	<i>Hd</i>
I	0.3 mi., Jan. 30	0.5 mi., Jan. 18	0.8 mi., Jan. 26
II	0.9 31	1.4 20	1.8 21

The amplitudes are consistently increased by the infusion of other workers; they may have observed objects brighter or different in kind. But the maximum phase remains nearly constant in date, about January (or April, July, October) 24.

7. It will probably appear that little weight attaches to the details of this analysis. There are resemblances in phase, but they may be little more than accidental. But one thing will be noticed. The dates of maximum phase point to months which are commonly rich in meteors. To test this idea the numbers of meteors contributing to the monthly means in Group I (Denning's data) have been taken from McIntosh's Table V and expressed in the form (5). The result is:

$$N = 69 + 27 \sin(t + 228^\circ) + 14 \sin(2t + 331^\circ) + 23 \sin(3t + 202^\circ) + 7 \sin(4t + 20^\circ). \quad (6)$$

The corresponding dates of maximum are August 28 for the first term, March 15 or September 15 for the second, April (August, December) 8 for the third and February (May, August, November) 2 for the fourth. It seems remarkable that these phases should agree as well as they do with the corresponding phases in the level. The main period in frequency agrees fairly in date with the mid-points of the paths. In the six-monthly term the greatest frequency precedes the highest level in general by a month, and in this agrees with *Ha* II. This is the greatest discrepancy, for the agreement of phase in both the third and fourth terms is close. On the whole there is a resemblance in detail between the variations in number and level which it is hard to attribute entirely to chance.

8. Evidently the correlation between the number of meteors and the monthly mean height is close. As an example, the correlation coefficient between *Hd* and number has been calculated for Group I. The result is $r = 0.713$. Superficially at least this correlation supports the idea that the seasonal variation in the height of meteoric phenomena is due to the cumulative effect of the meteors themselves. The meteors of which the observations have been used belong to the brighter class and are in themselves few, but they are no doubt accompanied by clouds of particles too small to be observed individually. The possible effect on the atmosphere need not be left out of consideration.

The alternative to a direct connexion of this kind between level and number is to suppose that both are related to some common seasonal change. There is a known correlation* between the height at which meteors become visible and their velocity, the coefficient being about $\frac{1}{2}$. The mass and nature of the meteor will also have their effects. Both velocity and substance have been suggested by Öpik as effective seasonal agencies for the variation in heights. It is clear that the number of possible factors is large and that they are connected in an intimate way. McIntosh has screened his material by the exclusion of the principal showers, but adds that "possibly a few members of these streams remain in the data." This may be an understatement. Certainly it seems that after their removal the remaining meteors which intersect the Earth's annual path form anything but a simple homogeneous aggregate unrelated in distribution,

* *M.N.*, 76, 391; *Probability and Frequency*, p. 242.

velocity and nature. Hence the weakness in Hoffmeister's derivation of the mean heliocentric velocity of meteors.*

9. One point needs to be further considered. The suggestion that the variations in the mean height depend on the number of meteors passing through the atmosphere, mentioned in § 8, can have no substance unless all the meteors passing near the time contribute to the effect. Now the numbers analysed in § 7 are those which remain after the principal showers have been removed. For the whole numbers or rates Denning's table is chosen out of a set of six given by Olivier.† As a matter of fact Schmidt's table but for a systematic ratio of 7/8 is very similar. The analysis of Denning's monthly rates (M) gives:

$$6M = 69 + 35 \sin(t + 203^\circ) + 13 \sin(2t + 18^\circ) \\ + 15 \sin(3t + 212^\circ) + 11 \sin(4t + 4^\circ). \quad (7)$$

The corresponding dates of maximum are: first term September 22, second February (August) 21, third April (August, December) 4 and fourth February (May, August, November) 6. The case, such as it is, is little altered by substituting the general for the "sporadic" rate. All that can be said is that the presence of meteoric matter in the atmosphere *may* have an effect in varying the height of meteoric phenomena.

On the other hand, the resemblance between the general and the "sporadic" rate is closer than could have been anticipated. It seems as if the removal of the shower meteors ought to have produced a far greater change than that shown in (6) and (7). For it means that the density distribution of the meteoric nodes round the Earth's orbit is much the same for the "sporadic" and for the shower meteors. In other words, the whole aggregate of meteors is one organic system, and that system can only be connected with the Sun. Such a view is certainly not without difficulty, because all the observational evidence points to hyperbolic velocities and a galactic, if not cosmic, origin. It seems hard to reconcile such an origin with the persistent distribution of the nodes which, it has been seen, is reflected even in some detail in the annual variation of the atmospheric level of terminal and intermediate points on meteoric paths. The meteoric system shows signs of structure, and it becomes an open question whether any meteors can properly be called "sporadic."

10. A consideration of the facts leads to a modified statement of one of the main problems. Radiants imply the existence of meteoric streams. These may be contracted, in which case the task of the observer is comparatively easy; or they may be diffuse, in which case the determination of the radiant sets a difficult problem, calls for skilled judgment and gives rise to much controversy. The path of the stream may be hyperbolic, for which there is much evidence in the observations of velocity, or it may be elliptic up to the parabolic limit. If it is hyperbolic a meteor ends its career in the Earth's atmosphere or, if it misses that fate, it passes by, never to return; in either event a continuance or repetition of the radiant implies that the stream is maintained by a continual supply of fresh material. If the stream follows a closed path the material near a given point may at different times give rise to the phenomenon of periodic showers. Some of these showers have been connected with periodic comets; more exactly the corresponding meteoric streams have been shown to follow the same, or nearly the same, paths as certain comets. The nature of this connexion is the problem in view. Presumably because the comet is the conspicuous object, it has been treated as the primary element and the meteoric stream as the derivative. But it seems possible that the order of importance should be reversed and the meteoric stream considered as the fundamental entity. Then the main question will cease to be, how are visible meteors derived from comets, which is probably not true, and it will take the form, how and in what circumstances does a meteoric stream constitute a comet? From this point of view some facts may be stated: (1) the same stream may form more than one comet,

* Cf. *M.N.*, 100, 316-317.

† *Meteors*, p. 182.

(2) a hyperbolic stream cannot form a comet at all, (3) a comet once formed may disappear, but the stream in some form must remain.

11. The subject of meteoric velocities has attracted much attention of late. Time was when it was commonly believed that the parabolic velocity represented in the main an upper limit. For this there was some excuse. The known connexion between meteor showers and cometary orbits was generalised perhaps too hastily. Then the work of E. Strömgren and of Fayet had removed the hyperbolic eccentricity from the orbits of comets in the initial form before planetary perturbations had taken effect. Accordingly there was a natural tendency to discount all exceptionally high velocities as the outcome of bad observations, the more readily as the estimates are difficult and demand the highest skill from the visual observer of meteors. It may therefore be well to point out that Denning's results do point to pronounced hyperbolic velocities in no uncertain way.

The Earth's mean orbital speed is $V_0 = 18.5$ mi./sec., the parabolic velocity is $V_0\sqrt{2} = 26.2$ mi./sec. and the mean square relative velocity is $V_0\sqrt{3} = 32.0$ mi./sec. Now the corresponding figure* found from Denning's data was about 34 mi./sec. The excess is not important. But (though this retracts a former argument) shower meteors with elliptic velocities must have entered into this result, and therefore to maintain the average on the parabolic level compensation must be found in the hyperbolic speeds of other meteors in the total. Moreover, meteors marked "swift" or "very swift" with no velocity recorded show that the computer recognized his limitations as an observer. It is natural to infer that if estimates of speed had been possible they would have been above the average and therefore of the hyperbolic order. It is curious that Denning's highest velocities, which probably were generally suspect at the time, are rather confirmed than otherwise by recent work. Speeds of the highest order present a similar problem, whether determined visually or with instrumental aid.

12. From the velocities determined by Denning the distribution †

$$dn = AV^3 e^{-0.131V} dV \quad (8)$$

was found to represent the relative speeds with rough approximation. It is not without interest to apply a formula of the same type to the heliocentric velocities given by Öpik.‡ With only one constant at disposal, apart from the number distributed, this will be taken to be

$$dn = \frac{100}{6 \times 14^4} V^3 e^{-V/14} dV, \quad (9)$$

or

$$\log \left(\frac{dn}{dV} \right) = 6.6372 - 10 + 3 \log V - 0.031V, \quad (10)$$

the unit of V being km./sec., whereas the unit in (8) is mi./sec. The result of the comparison follows.

TABLE II
Heliocentric Velocities of Meteors

	12.5	14.9	17.7	21.1	25.1	29.8	35.4	42.1	50.1	59.6	km./sec.
O	1.1	0.6	0.2	2.5	3.1	9.6	16.8	19.4	15.6		per cent.
C	1.0	1.6	2.7	4.1	6.0	8.2	10.6	12.7	13.6		"
O - C	+0.1	-1.0	-2.5	-1.6	-2.9	+1.4	+6.2	+6.7	+2.0		
	59.6	70.9	84.3	100.2	119.2	141.7	168.5	200.5	238.3	283.3	km./sec.
O	11.7	9.8	5.7	2.1	0.3	0.4	0.5	0.5	0.1		per cent.
C	12.9	10.7	7.5	4.3	2.0	0.8	0.2	0.03	0.003		"
O - C	-1.2	-0.9	-1.8	-2.2	-1.7	-0.4	+0.3	+0.5	+0.1		

* *M.N.*, 76, 394, 1916.

† *M.N.*, 76, 393, 1916.

‡ *M.N.*, 100, 319, 1940.

13. The result of this comparison is not very good, nor is there any reason to expect anything much better. The formula seemed to represent in a rough way Denning's results, which are apparent speeds relative to the Earth. The same form is now applied to heliocentric velocities, and it is not surprising that the representation is far from good, notably in the neighbourhood of 40 km./sec. Nevertheless the original material has not been given in such an informative shape but that the formula (9) may have a slight use. Thus it places the mode at 42 km./sec. and gives an average speed of 56 km./sec. and the mean-square velocity of 63 km./sec. These figures seem to represent the material quite fairly.

The corresponding results from Denning's material are 22.8 mi./sec. = 36.5 km./sec., 30.4 mi./sec. = 48.6 km./sec., and 34.1 mi./sec. = 54.6 km./sec. That is to say, they are distinctly smaller, about $\frac{6}{7}$ in fact, whereas referring as they do to relative velocities, they should apparently be larger. Denning's material contained a greater proportion of large bright meteors, and on that account an explanation of the discrepancy, if established, does not seem to be out of the question.

On the other hand, the Arizona observations used by Öpik, though made with instrumental aid (the rocking mirror), are far from being as satisfactory as could be wished. The accidental error is admittedly high and can be reduced with greater experience. The heights were so poorly determined that mean heights were substituted. In the last stage, the reduction to heliocentric velocities, a hypothetical element enters, the effect of which cannot be gauged. On the whole, at the present moment there may be some reason for satisfaction that the workers who have more recently entered this field of observation seem to have obtained results of at least the same order of magnitude as their predecessors.

14. Constructive criticism tending to improve the quality of observations is always to be welcomed. But disparaging criticism, which may have the effect of reducing the quantity of visual observations, is to be deprecated. For our present knowledge of meteors has been due almost entirely to the work of visual observers. Nor is that position likely to be changed quickly. Special researches with instrumental help, such as spectroscopic and photometric studies, will be most valuable. Photography may introduce a higher standard of accuracy. Well-organized expeditions may serve to focus attention on special problems and enlist the enthusiasm of young observers. All these things will be to the good. But though they can supplement, they can never replace the steady persistence of the experienced visual observer. If there was a weakness in the position it was one fully recognized by such a man as Denning. It was not the difficulty of attaining a sufficient degree of accuracy, given the necessary aptitude and assiduity, but the need for organized co-operation by a larger number of qualified workers in this arduous field.

To take a simple example, the distribution of meteors through the year is a fundamental problem. Olivier* gives the monthly frequencies according to six observers. His own list places May and June among the richest months, whereas the other five observers agree that these are among the poorest. The discrepancy is attributed to the fact that 1916 was an exceptional year. The lesson is clear. In order to be of value, counts must be continued in uniform conditions over long series of years. Something has been done to extend this simple but arduous research to other latitudes, but much remains to be done and final success will always be in jeopardy unless the co-operation of many observers in all parts of the world is assured. If the prevalence of hyperbolic velocities of galactic origin is established, the influence of the solar motion ought to become much more conspicuous than it has been shown to be hitherto. In the meantime the origin of "sporadic" meteors remains an unsolved and highly interesting question.

So it is with the determination of radiant, the location of the paths of meteors in the atmosphere and the timing of their motion. For all these things the collection of

* *Loc. cit.*

material in adequate quantity must rest with the visual observer who is prepared to compensate for his poverty in instrumental resources by his personal skill and zeal.

Summary

1. The relative merits of the arithmetic and the harmonic mean for the heights of meteors are discussed.

2. The seasonal variation in the heights of meteors at three points in their observed paths is submitted to harmonic analysis.

3. The corresponding number of observations is analysed in the same way. It is found that the monthly numbers are highly correlated with the corresponding mean heights.

4. When the general monthly rate of meteors is analysed the result does not differ greatly from that obtained after the shower meteors have been excluded. This persistence in the distribution of meteoric nodes suggests a pervading structure of the whole system and leads to other questions.

5. The distribution of meteoric velocities is considered. The occurrence of hyperbolic velocities is inferred from Denning's determinations no less than from recent observations.

6. Stress is laid on the value and importance of the visual observation of meteors.

ON THE LIMITS OF UNIFORM GALACTIC ABSORPTION

George Alter, D.Sc.

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Many investigations in the last decade have been devoted to the question of general galactic absorption, but the results are rather discordant, partly because of the recognized irregular distribution of the absorbing matter, partly because of uncertain distances of the stars, and partly because of the non-homogeneous working material which includes early stars, late stars, Cepheids, open clusters, globular clusters, planetary nebulae, extragalactic nebulae and star distribution. The space reddening in any region of the sky is due either solely to the influence of an interposed dark cloud, or to a combined influence of a cloud and uniformly distributed absorbing matter in this sector or throughout the Galaxy, or, finally, to this uniform layer of absorbing matter alone. The absence of this selective absorption in any region does not, however, imply the absence of absorption, for it becomes zero also when the photographic and visual absorptions are equal, however large they may be. It is therefore important to establish whether there is a relation between the total and the selective absorption; the constant part of this relation would constitute the contribution of the uniform layer. The simplest form of this relation is assumed to be the ratio between the total photographic and the selective absorption. But the values of this ratio in different investigations show also a large scatter. Schalén * found from early stars in different sky regions a mean value of 10, Müller and Hufnagel † from CI in the North America cloud 5 to 6, Oort ‡ from dark and bright nebulae 10 to 12, Stebbins, Huffer and Whitford § from B stars 4.5 (or 9 on the photo-electric scale), Trumpler || from open clusters 2, Hunter and Martin ¶ for the ratio m_{pv} /gradient excess from early stars 2, and Seares ** from extragalactic nebulae 10.

It is, however, possible to find theoretically the probable limits of the relation between the photographic and the selective absorption. If M_1 is the photovisual absolute magnitude derived from the observed CI with the help of the tables of King †† and Lundmark †††, and r_1 the distance computed by formula (1) below, M_0 and r_0 the corresponding values got by the spectral types, m_{pv} the observed photovisual magnitude, and a_{pv} , a_p and a_s the photovisual, photographic and selective absorption coefficients in magnitudes per kiloparsec, respectively, then

$$m_{pv} - M_1 = 5 \log r_1 - 5, \quad (1)$$

$$m_{pv} - M_0 = 5 \log r_0 + a_{pv}r_0 - 5. \quad (2)$$

Subtracting (1) from (2) we get

$$M_1 - M_0 = 5 (\log r_0 - \log r_1) + a_{pv}r_0. \quad (3)$$

The term on the left side of equation (3) can be derived from the curve M against CI (fig. 1). This curve can be represented in the average by a straight line, covering all

* *Medd. Upsala*, 58, 36, 1934.

† *Ann. d' Astroph.*, 1, 85, 1938.

‡ *Ap. J.*, 91, 186, 1940.

** *P.A.S.P.*, 52, 108, 1940.

†† *Ibid.*, V, 539, 1932.

† *Zs. f. Ap.*, 9, 357, 1935.

§ *Ap. J.*, 90, 213, 1939; 91, 25, 1940; 92, 195, 1940.

¶ *M.N.*, 100, 674, 1940.

†† *Handbuch für Astrophysik*, V, 173, 1932.

spectral classes. (It may be mentioned that perhaps the shape of the curve would be altered if the absolute magnitudes were redetermined, taking into consideration the

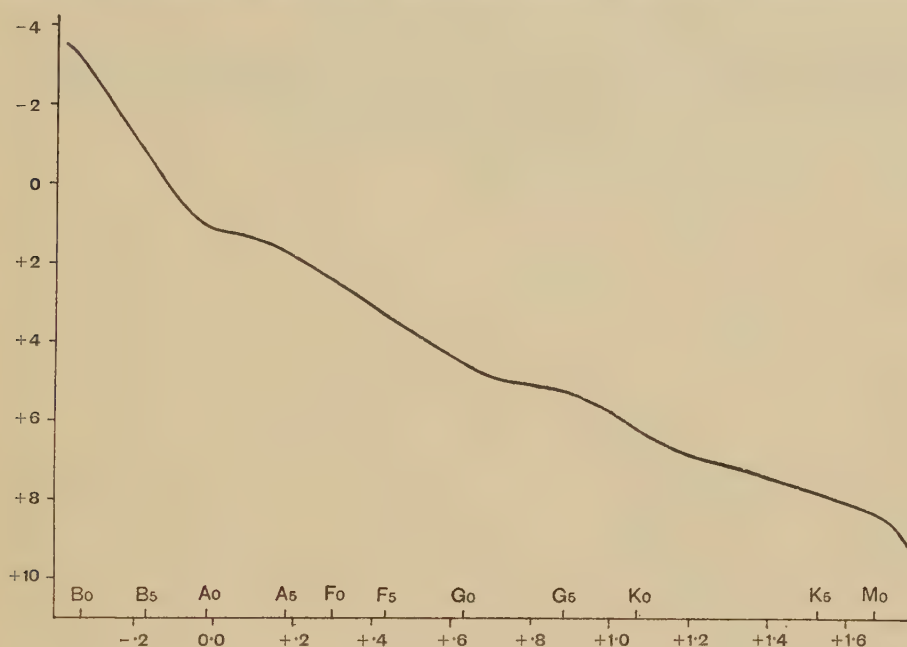


FIG. 1.— $M - CI$ curve.

absorption, as already suggested by Stebbins, Huffer and Whitford *, but it is unlikely that the mean slope would suffer important change.) Then

$$M_1 - M_0 = k a_s r_0, \quad (4)$$

where k is the slope of the straight line and $a_s r_0$ is the colour excess, or the change in CI corresponding to the change in M . Combining (3) and (4) we get finally

$$\log r_1 = \log r_0 + b r_0, \quad (5)$$

where

$$b = 0.2(a_{pv} - a_s k). \quad (6)$$

Equation (5) represents a family of curves dependent on the parameter b (fig. 2), and the approximation of these curves to observation will determine the probable range of b . The curves show a maximum for negative values of b (for positive values of b there is a minimum in the unobservable part of the curves with negative distances) and an inflexion point. The maximum occurs at

$$r_1 = -\frac{\log e}{eb}, \quad r_0 = -\frac{\log e}{b}. \quad (7)$$

The presence of such a maximum means that we cannot establish by observation (using formula (1)) a larger r_1 for a given b , and that any other observable r_1 is bivalent for the determination of r_0 . Experience shows, however, that there are values of r_1 which can be determined by (1), and the largest value found must exclude all values of b larger than that found by inserting this r_1 in (7), thus setting a probable limit for the negative values of b . The equations (7) give a straight line with a slope of $1/e$ for the relation

* *Ap. J.*, 90, 214, 1939.

between r_0 and the maxima of r_1 , represented by the dotted line in fig. 2. The bivalence of r_1 is of no practical importance since the second values give distances r_0 , making observation improbable.

For positive values of b the slopes of the curves increase with increasing b in such a way that, after a certain point, even a small gain in r_0 corresponds to an incredibly high increase of r_1 . This gives the probable limit for positive values of b .

For $b=0$ we get a straight line. In this case $a_{pv}=ka_s$ and $r_1=r_0$, i.e. if the ratio

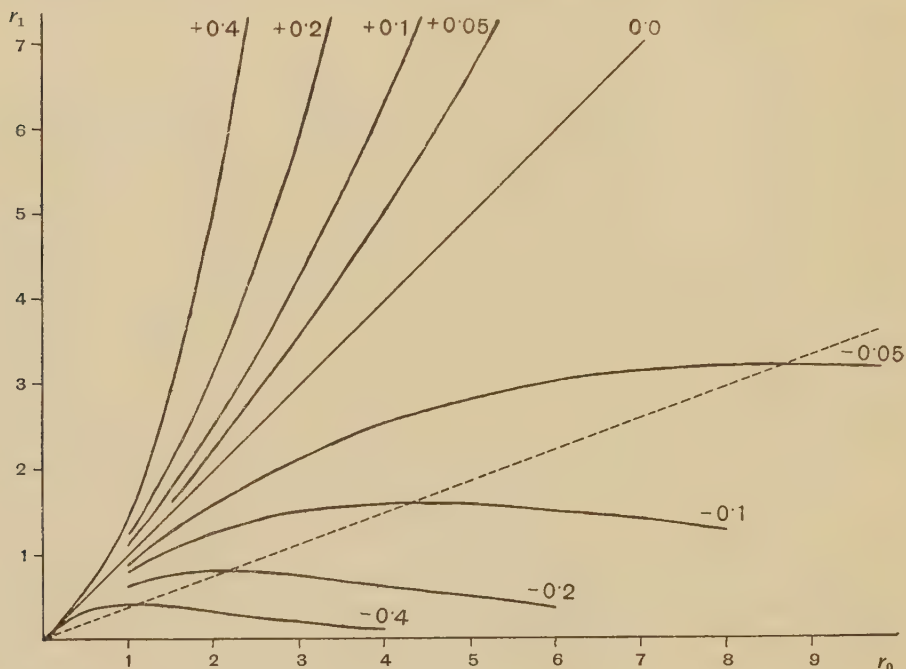


FIG. 2.—Relation between r_1 and r_0 with changing b .

between the photovisual and the selective absorption coefficient is equal to the slope of the $M-CI$ curve, then the observed distance is equal to the true one.

An examination of fig. 2 shows that the range of b is limited by about -0.1 , because values of r_1 up to 1.6 kiloparsec (the maximum of the curve with $b = -0.1$) have evidently been observed, and $+0.1$, because increases of r_1 above $b = +0.1$ get improbably high.

The relation between the photographic and the selective absorption is given by (6) after putting $a_p = a_{pv} + a_s$. Then

$$a_p = k_1 a_s + 5b, \quad (8)$$

where $k_1 = k + 1$. The mean value of the slope of the $M-CI$ curve gives $k_1 = 7.5$.* The relation between a_p and a_s must, however, be valid for the blue stars with a sharp slope and for the red ones with a shallow slope; the respective values are $k_b = 14$ and $k_r = 5$. When we insert these values in (8) and shift the straight line, represented by this equation, along the values of b between -0.1 and $+0.1$, we get two bands each limited by the straight lines corresponding to the limiting values of b (see fig. 3). Within these two bands every point gives the respective a_p and a_s suitable for the corresponding k_1 . The true values for any k_1 are therefore contained in the quadrangle belonging to all bands. Its corners are A, B, C, D. The line AD has to replace AD' because the triangle ADD'

* Fig. 1 does not give the right slope because the rate of the ordinate is only one-tenth of that of the abscissa.

represents the points with $a_s > a_p$, which is impossible. AD is the line giving the lowest possible limit (unity) of

$$f = a_p/a_s = k_1 + 5b/a_s, \quad (9)$$

thus representing an absorbing matter with absent a_{pv} . AB, on the contrary, gives $f = \infty$, representing an absorbing matter with absent a_s , or $a_p = a_{pv}$. BC gives the limit for $b = +0.1$ and CD for $b = -0.1$, and the intermediate values of f .

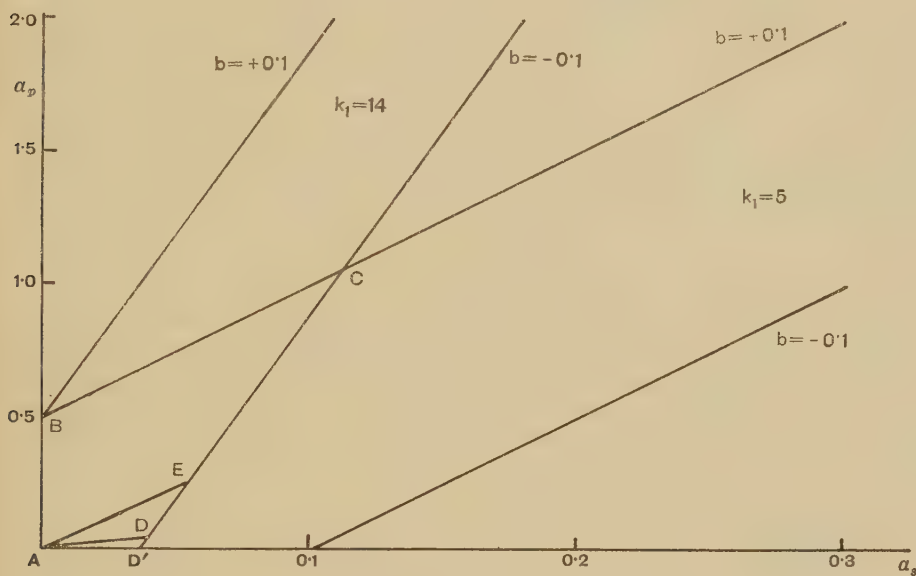


FIG. 3.—Relation between a_p and a_s .

The influence of the dispersion of the $M - CI$ curve (fig. 1) has no effect as long as the slope remains within the extreme limits of k_b , and k_r , because in this case the size of the quadrangle would not be changed. But it might happen that the dispersion would slightly increase the upper and decrease the lower limit. Then the slopes of DC and BC (fig. 3) will change, respectively, in the same way, and the quadrangle will become still smaller. The inclusion of giants in this investigation gives some difficulties because there are not enough data to draw a $M - CI$ curve and to determine thus a slope. If we suppose, as is usually done, the same absolute magnitudes for all giants, then the absolute magnitude is not changed by a CE, $M_1 = M_0$ and $k = 0$. Then the lower limit of k_1 is not 5, as shown in fig. 3, but 1, and the quadrangle becomes smaller again by decreasing the slope of BC.

The assumptions of the above investigation, namely, the validity of the $M - CI$ curve for the parameter k and the distance conditions giving the limits of the parameter b , are based on experience. These two assumptions determine the slopes and the widths of the bands in fig. 3 and thus the size of the quadrangle ABCD. If we use as a further assumption, a theoretical one, the selective law, derived again by Stebbins, Huffer and Whitford * and resulting in $f = 4.5$ on the international scale, we are able to restrict the number of suitable points within the quadrangle to one of the points on the line AE with a slope of 4.5. Inserting this value in (9), b becomes negative and the extreme values are $a_p = 0.25$, $a_s = 0.055$, represented by the point E. The relatively small area covered by the admissible values of the possible points explains why it has been impossible to prove the presence of a continuous and uniform absorbing galactic layer. All the values of absorption coefficients at present determined, as far as I am aware,

* *Ap. J.*, 90, 212, 1939.

lie outside this region, and must therefore be explained by their derivation from irregular absorbing clouds.

The decision as to the final position of the point can, of course, be found only by further observation, and it may therefore be suggested that the absorption conditions in the "galactic windows" might be carefully investigated, thus avoiding as far as possible the influence of absorbing clouds. In these regions the uniform layer might occur in its purest form and in sufficient depth.

Another investigation is suggested by the shape of the b curves (fig. 2) for negative values of b . The interval Δr_0 becomes larger for the same interval Δr_1 when approaching the maximum. Assuming a uniform spatial distribution of stars in r_0 , this increase of Δr_0 corresponds to an increase of the spatial star density in r_1 when approaching the maximum, and to a density maximum corresponding to the curve maximum itself. It may be mentioned that Seares * derived, by the help of stellar statistics, a similar result, showing that the star density reaches a maximum at a certain distance, with a constant absorption coefficient.

Summary

The present paper shows that the absorption conditions in a uniform and continuous absorbing galactic layer can be determined by the observed relation between colour index and absolute magnitude, and limits are found for the absorption coefficients. An assumption on the proportionality between the photographic and the selective absorption coefficient restricts these limits further. Suggestions are made for the determination of the actual coefficients.

Norman Lockyer Observatory,

Sidmouth :

1940 December 2.

* *Ap. J.*, 74, 98, 1931.

A PHOTOGRAPHIC SURVEY OF GALACTIC CLUSTERS:

II. N.G.C. 663, 659, 654, 559, Tr. I, Appendix to N.G.C. 581

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The method of this investigation was described in an earlier paper * (designated later on as A I), and it was mentioned that it is limited by the lack of knowledge of spectral types for the cluster stars, a knowledge which decides not only the division in dwarfs and giants but also the true CI. In the meantime a paper of R. S. Zug † has become available in which the spectral types for a number of stars in 23 galactic clusters are quoted from an unpublished investigation of Trumpler. This permits an extension of the method for these clusters by the determination of true absolute magnitudes M_0 and of the colour excess (CE), leading to an improvement of the distance results. The change in M against the value M_1 got by the observed CI (King's table ‡ and Lundmark's table, last column §) alters the order of $m - M$, the range within which the stars are assumed to belong to the cluster, and, finally, the average value of $m - M$ determining the distance. The change of the range may result in a change of the number of cluster stars. An indication of the certainty of this value is the dispersion around the statistical value n got by the counting of cluster stars. The dispersion, or "natural uncertainty" ||, $\sigma = n^{\frac{1}{2}}$, limits the difference between the two values if they are fairly accordant. The distance determination with the help of the average $(m - M_1)$ value is amended by the assumption of an absorption where a CE is present. In this case the distance r is given by the formula

$$m - M_0 = 5 \log r + a_{pv}r - 5, \quad (1)$$

where a_{pv} is the absorption coefficient for photovisual light; r can easily be plotted with a given $m - M$ on a curve drawn with a constant a_{pv} . This, of course, involves another assumption, namely, that the density of the absorbing matter is constant and that the distribution of this absorbing matter is uniform in the galactic plane, for the value of a_{pv} was determined under this assumption. This assumption seems, however, to be valid only in a statistical sense.

Some investigations of star numbers and of B-type stars have shown the occurrence of absorbing clouds with different absorbing qualities in different regions, the unequal distribution of absorption in different galactic longitudes, and an apparent lack of absorption in the so-called "galactic windows" with the help of extragalactic nebulae.

The use of a constant value of a_{pv} has therefore only a limited certainty, and must be abandoned when low or absent absorption is proved or indicated by other means. The value of $a_{pv} = 0^m.35$ per kiloparsec, used with these restrictions in the following investigation, is taken from Trumpler's ¶ paper on galactic clusters. The choice of Trumpler's value among many others referred to by Bok ** is due to its derivation from the same kind of material as the present.

Another indication of absorbing matter may be given by the background densities near the clusters. They must, however, be treated very cautiously, because they are

* M.N., 100, 387, 1940.

† L.O.B., 16, 119, 1933.

‡ Handbuch für Astrophysik, V, 173, 1932.

§ Ibid., V, 539, 1932.

|| Bok, The Distribution of the Stars in Space, 3, 1937.

¶ L.O.B., 14, 167, 1930.

** Loc. cit., 112, 1937.

also dependent on the distribution of stars in space. Here, too, the treatment can be only statistical. A comparison of the CE of 15 of Zug's clusters with the background densities of these clusters in Raab's * survey shows a large scatter, as was to be expected, partly because of the inadequacy of the magnitude range in the two investigations. Still, there is statistically a decrease in the density with increasing CE. There is a possibility of diminishing the scatter when we consider the co-ordinates of the clusters. The distribution of the clusters shows two separated groups with respectively higher and lower background density. In the higher group of 9 clusters 4 belong to the Puppis region and 3 to the Cassiopeia region; of the 6 clusters of the lower region 4 belong to the Auriga region. Table II in Greenstein's † paper on the absorption indicates absorbing clouds at distances of 500 and 600 parsecs for the first two regions, respectively, while in the third the distance is only 145 parsecs. Consequently, the Auriga region is more obscured (for foreground stars) than the other two and the density is lower. When we shift the Auriga clusters in the density co-ordinate to the upper group to make them more homogeneous, the scatter becomes much smaller and the relation between CE and density more pronounced. A similar relation was found in the comparison of four of Zug's CE with the background densities in the two papers of H. Klauder and H. Siedentopf ‡ on star numbers, the scatter being very much smaller. The difference of the scatter is mostly due to the difference in counting: Raab counted the stars on the Franklin-Adams Charts, while Klauder and Siedentopf used several plates exposed for this purpose.

Another indication of absorbing matter is given by the range of the distance moduli of star clusters. The $(m - M)$ range of the cluster stars is dependent on the error of spectral classification (or CI) and the natural scatter of absolute magnitudes (about $\pm 0^m.5$). It is the arithmetical sum of these two errors, expressed in magnitudes. But the range is also dependent on the spectral class (Sp) itself. The reduction of the Sp-error to M -error is done from the $M - Sp$ curve, § and it is obvious that the same Sp-error gives varying M -errors corresponding to the varying slope of the $M - Sp$ curve. The slope (or the change in M) for B stars is much larger than for A and F stars, and the slope for these stars is larger than for G and K stars. Thus the $(m - M)$ range decreases—for the same Sp-error—(generally) from B- to K-type stars. The same is valid for a CI-error, provided that the CI agrees with the spectral type, because the true absolute magnitude is determined from the true spectral class. The reduction is done with the help of a $M - CI$ curve. Thus, for instance, the CI-error of the present investigation ($\pm 0^m.18$) corresponds to a M -error of $\pm 2^m.0$ to $2^m.5$ for B stars, $0^m.8$ to $1^m.2$ for A and F stars and $0^m.5$ to $0^m.9$ for G and K stars, and the corresponding $(m - M)$ ranges should be 5^m to 6^m , $2^m.5$ to $3^m.5$ and 2^m to 3^m , respectively. If, however, there is any absorption, the B stars, for instance, appear reddened, and the expected range, reduced from the falsified CI, is smaller than it should be and than it actually appears in the observed value. An example of this case will be found in the present paper. N.G.C. 663 shows a range corresponding to B stars (5^m to 6^m), contrary to the observed CI (range $2^m.5$ to $3^m.5$). Thus the range itself indicates that the CI is not right, and the CE of $0^m.63$, established with the help of Sp, confirms the absorption. The same occurs with N.G.C. 654, where the range does not correspond to the average CI. The situation of this cluster in a dark region, as mentioned later, suggests an absorption, a suggestion which is supported by the indication of the range. Thus we can assume that an irregular behaviour of the $(m - M)$ range indicates the occurrence of absorbing matter.

A further reason for the range scatter is the estimation of range limits. The range limits within the $(m - M)$ series of all region stars (including foreground and background stars) are determined by the appearance of a nearly continuous sequence of small steps

* *Lund Medd.*, Ser. II, 28, 1922.

† *H.A.*, 105, 3 60, 1936.

‡ *A.N.*, 265, 209, 1938; 268, 197, 1939.

§ See author's paper on Galactic Absorption, page 84 (fig. 1).

between the individual $m - M$ against the larger ones of the fore- and back-ground stars. If the cluster is well detached from the other stars in the line of vision it is quite easy to establish this limit, as in N.G.C. 559. If it is not well detached, there will be some stars so near the limits that it is doubtful if they belong to the cluster because the steps show no jump; in this case the estimation of the limit is not so certain as in the former one. A closer examination of the relations between range and CI, and range and absorption, is intended when more clusters are available, but the present paper shows a good agreement with the results mentioned above.

The clusters of this paper, together with N.G.C. 581 described in A I, are all situated within a relatively small region of Cassiopeia of about 6 sq. degrees. The star distribution in this area has been investigated by Schalén *, who attributes the irregularities in the distribution to real differences in space distribution, while Bok † assumes in addition several patches of dark nebulae. N.G.C. 581, Tr. 1 (one of the anonymous clusters which Trumpler added to his catalogue), and N.G.C. 659 lie within the tongue-shaped bright region in fig. 13 of Schalén's first-mentioned paper, N.G.C. 559, 654 and 663 in the dark region north of it. The average background densities per sq. degree are respectively 3935, 3253, 3269 and 2979, 1599, 2595. The difference between the average of the two groups may raise the suspicion of absorption, which is, moreover, proved for N.G.C. 663 by the CE. A closer examination of the star field in this region shows that N.G.C. 654 is in the centre of a specially dark square of about $1^{\circ}.5$ width ‡, on the southern boundary of which N.G.C. 663 is situated. The results of N.G.C. 654 will therefore be corrected by assuming the corresponding absorption.

N.G.C. 663

$$\text{R.A.} = 1^{\text{h}} 39^{\text{m}}.2, \quad \text{Dec.} = +60^{\circ} 44' (1900.0)$$

This cluster is mentioned in the catalogues of Shapley §, Trumpler || and Collinder ¶, and was investigated by Gushee **, Raab ††, Wallenquist ‡‡, Schalén §§, Trumpler ||||, C. J. Anger ¶¶, Zug ***, C. Anger Rieke †††, Klauder and Siedentopf ††† and Sticker. §§§

The present investigation was made on the same set of plates as in A I, and the magnitudes are therefore subjected to the same reductions.

The star-counting gave the total number of stars in the cluster region to be 300. A comparison with Wallenquist's catalogue showed that 17 more star images are hidden behind those of bright stars, the telescope being unable to separate them. Having corrected for this reason the star number from 300 to 317, the distribution of stars within the cluster region and its surroundings results in a background density of 2595 stars per sq. degree and in 165 cluster stars within a cluster region of $16'.4$ diameter. Fig. 1 gives the diagram of the star density per sq. degree within the cluster region and its surroundings. The density distribution agrees fairly well with the results of Klauder and Siedentopf, who show that this cluster lies on the boundary of an absorbing cloud. A decrease in density in the ring zone of about $20'$ distance from the centre is found also in the present investigation. The density distribution in different polar angles around the cluster, discussed by Klauder and Siedentopf, involves the two clusters N.G.C. 654 and 659 (dealt with in this paper later), which are situated within

* *Medd. Upsala*, 37, 66, 1928; 45, 1929; 55, 43, 1931.

† *Loc. cit.*, 76, 1937.

‡ See also Ross Calvert's *Milky Way Atlas*, Plate 26, about $1\frac{1}{2}$ in. below the centre.

§ *Star Clusters*, 1930.

|| *L.O.B.*, 14, 170, 1930.

¶ *Lund Annals*, II, 1931.

** *A. J.*, 32, 117, 1919.

†† *Lund Medd.*, Ser. II, 28, 1922.

‡‡ *Medd. Upsala*, 42, 1929; *Ann. Lembang*, IV, 5, 1933.

§§ *Ibid.*, 49, 1930; 53, 1931.

|||| *L.O.B.*, 14, 156, 1930.

¶¶ *H.B.*, 883, 36, 1931.

*** *L.O.B.*, 16, 119, 1933.

††† *H.C.*, 397, 1935.

††† *A.N.*, 268, 199, 1939.

§§§ *Zs. f. Ap.*, 1, 174, 1930.

the investigated region. Fig. 13 (2) in their paper shows that the background density in the sector containing N.G.C. 659 (polar angle 340°) is higher, and that containing N.G.C. 654 (polar angle 210°) lower, than that of N.G.C. 663. The results of the present investigation give accordingly 3269 and 1599 respectively, against 2595 for N.G.C. 663.

A table of the CI with increasing distance modulus $m - M_1$ shows that, among 118 stars with determined CI, 100 belong to the cluster range of $m - M_1$, and that this range, however, is unusually extended between 4.5 and 13.9; the average distance modulus is 9.2 and the distance 700 parsecs. This extension of the range indicates that the values of M , derived from the observed CI, do not agree with the true value. This can be

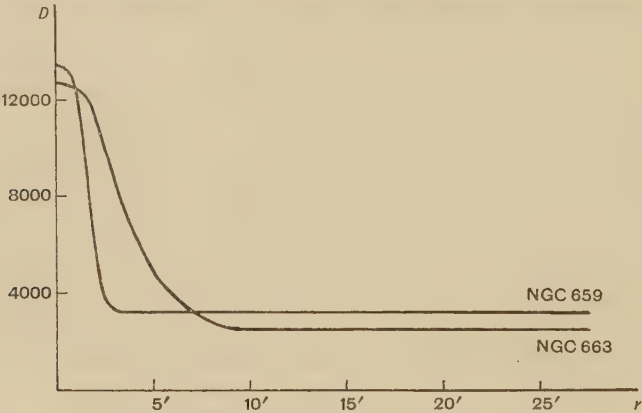


FIG. 1.—Star density per sq. degree.

proved for 28 stars for which a determination of spectral type is available. These spectral classes permit, moreover, the determination of the CE. Table I gives both $m - M_0$ and CE, using Trumpler's spectral types published in Zug's paper. Stars 73 and 199 apparently do not belong to the cluster. The average $m - M = 11.3$ and the distance from formula (1) is $1450 (\pm 100)$ parsecs. The mean CE is $+0.62$.

TABLE I
Distance Modulus and Colour Excess (N.G.C. 663)
 Z = Zug's star number

Star	Z	$m - M$	CE	Star	Z	$m - M$	CE	Star	Z	$m - M$	CE	Star	Z	$m - M$	CE
73	11	7.3	-0.22	207	23	10.7	+0.84	151	24	11.2	+0.56	204	8	11.8	+0.37
199	12	7.3	+0.38	172	41	10.8	+0.79	54	7	11.3	+0.97	76	*	11.9	+0.93
84	26	10.0	+0.34	211	17	10.8	+0.75	49	20	11.4	+1.15	108	13	11.9	+0.89
148	29	10.5	+0.23	173	15	11.0	+0.54	166	35	11.5	...	157	1	11.9	+0.19
279	6	10.5	+0.91	189	5	11.0	+0.83	224	18	11.5	+0.84	101	*	12.0	+0.63
181	32	10.6	+0.83	163	33	11.1	+0.67	143	25	11.6	+1.01	179	9	12.0	+0.82
182	31	10.7	+0.37	185	45	11.1	+0.33	265	19	11.6	-0.23	96	28	12.2	-0.10

* 2 or 4 which have by mistake in Zug's catalogue the same co-ordinates.

In order to extend this investigation statistically to the other CI stars of this cluster, we have to consider the magnitude range over which it is valid. Spectral types and mean m_{pv} of the stars of Table I give the following array:—

B1	B3	B4	B5	B6	B8	A0	A2
9.11	9.77	9.92	10.72	11.01	11.55	11.12	11.98

This increase in m_{pv} of about 3^m with changing spectral class corresponds fairly well to an increase in M_0 of -2.8 to $+1.3$, a further confirmation that these stars are at the same distance. The CE shows, however, a decrease with a similar change in spectral classes. (Such a decrease appears also in Zug's CE.) The mean CE of the spectral types B1 to B6 is 0.69, of B8 stars it is 0.64, dropping to 0.57 after including the two single A stars. The CE decrease of 0.05 in the B stars is probably due to the "magnitude excess." Hunter and Martin* notice that the effect in early B-type stars is small, and their Table IV gives for the type range O9-B7 the coefficient $c = -0.005$. This results, for the magnitude range of 4^m , in a CE decrease $\Delta CE = c\Delta m = 0^m.02$ against $0^m.05$ for nearly the same spectral region in the present case. The small difference is due to the uncertainty of the determinations, but the negative sign of c is confirmed by the present result. The statistical application of the mean CE is restricted in the present paper to stars within Trumpler's magnitude limits, *i.e.* down to $12^m.0$, and a CE of 0.69 is used for stars down to $11^m.0$, and $0^m.57$ for stars between $11^m.0$ and $12^m.0$. For fainter stars no comparison with Trumpler's results is available, and no mean values of CE can be assumed. In Table II M is derived from the CI corrected by the above values of CE.

TABLE II

Co-ordinates, Magnitudes and Distance Moduli of 54 Stars (N.G.C. 663)
Zero point = star number 157 (B.D. 60° 339), unit = 1'.0, X = east, Y = south
Co-ordinates of the assumed cluster centre X = -0.2, Y = +3.3

Star	X	Y	m_p	m_{pv}	$m - M$	Star	X	Y	m_p	m_{pv}	$m - M$
73	-3.3	-1.2	10.02	9.88	7.3	163	+0.4	+0.8	12.54	11.94	11.1
199	+1.5	+3.1	10.39	9.71	7.3	185	+1.0	-1.4	12.24	11.99	11.1
165	+0.4	+1.2	12.46	11.43	8.0	151	-0.1	+2.1	11.51	11.09	11.2
36	-5.0	+7.8	12.25	11.30	8.4	54 ‡	-3.9	-3.4	10.19	9.46	11.3
239	+3.1	+3.6	12.60	11.59	8.4	49	-4.3	+1.0	11.11	10.17	11.4
8	-7.5	+2.6	12.66	11.66	8.5	224	+2.3	+2.1	10.90	10.27	11.5
164	+0.4	+1.4	12.75	11.74	8.5	262	+4.6	-2.4	11.84	11.40	11.5
215	+2.0	-1.9	12.38	11.46	8.8	50	-4.1	+6.4	11.84	11.37	11.6
19	-6.1	+1.1	11.21	10.37	8.9	101 §	-2.2	+1.9	9.54	9.18	11.6
93	-2.5	-2.8	10.70	10.06	9.2	143	-0.6	-0.3	11.62	10.69	11.6
61	-4.0	+8.3	11.05	10.41	9.5	265	+4.7	+3.0	10.82	10.42	11.6
98	-2.2	-0.9	11.33	10.65	9.6	204	+1.6	+6.3	10.09	9.96	11.8
41	-5.2	+2.9	11.85	11.17	9.9	47	-4.3	+4.9	12.21	11.77	11.9
159	+0.2	+1.4	12.50	11.75	9.9	76	-3.2	+1.6	9.69	9.05	11.9
28	-5.3	+4.1	12.46	11.70	10.0	108	-1.8	+3.8	10.75	10.10	11.9
84	-2.8	+3.5	11.44	11.12	10.0	157 ¶	0.0	0.0	9.01	9.11	11.9
139	-0.8	+7.7	11.00	10.41	10.1	179 **	+0.8	+5.4	10.73	10.15	12.0
234	+2.8	+3.3	12.03	11.43	10.2	198	+1.5	+1.7	11.97	11.55	12.0
279 *	+5.6	+3.1	10.03	9.33	10.5	263	+4.5	+9.7	12.40	11.96	12.1
148	-0.4	+1.2	11.56	11.41	10.5	96	-2.4	+2.4	11.31	11.58	12.2
181	+0.9	+0.9	12.25	11.50	10.6	242	+3.2	-0.7	11.40	10.93	12.3
207	+1.8	+3.8	11.28	10.58	10.7	121	-1.4	+6.2	11.75	11.37	12.4
182	+0.9	+2.5	12.27	11.98	10.7	11	-6.9	+6.3	12.27	11.87	12.5
172	+0.7	+1.6	12.46	11.75	10.8	276	+5.3	+7.3	11.30	10.88	13.0
211	+1.9	+1.2	10.73	10.15	10.8	184	+0.9	+7.6	10.51	10.36	14.4
189 †	+1.3	-0.4	9.76	9.17	11.0	203	+1.6	+8.1	10.87	10.67	14.7
173	+0.7	+3.0	10.79	10.42	11.0	298	+7.6	+4.7	11.40	11.42	15.4

* = B.D. 60° 347. † = 343. ‡ = 329. § = 333. || = 331. ¶ = 339. ** = 340.
Star 189 = Burnham 911; Burnham 909 cannot be identified.

* M.N., 100, 656, 1940.

The table gives an average distance modulus of the cluster group $m - M = 10.79$, and consequently formula (1) determines distance r and linear diameter d :

$r = 1200 (\pm 100)$ parsecs, $d = 16'.4 = 5.7$ parsecs.

Among the 54 stars of Table II 49 belong to the modulus range. The same relation within the 118 CI stars gives 107 of them in the modulus range. An investigation of a nearby region results in 54 as the number of background stars within the same area and modulus range, and this number must be subtracted from the value 107 to give the number of true cluster members amongst the CI stars. The ratio between the remaining 53 and 107 determines the number of cluster stars amongst the total count of 317 to be 157. The difference of 8 between this value and the 165 got by statistical counting (mentioned above) lies within the dispersion $\sigma = \pm 13$. The magnitude distribution is given in the following table:—

m_{pv}	10	11	12	13	> 13.5
Number of stars	9	11	8	13	116

The results of previous investigations were:

	Distance Parsecs	Diameter	
			Parsecs
Shapley	790	11	2.5
Trumpler	2170	14	7.4
Collinder	2500	15	10.9
Raab	1000	18-22	5.2-6.4
Wallenquist	790	22-25	5.1-5.7
Zug	1750		
Schalén	2460		
Anger Rieke	2130		

Star catalogues have been published by Gushee, Wallenquist, C. J. Anger and Zug. Table III gives a comparison between the magnitudes and CI of the present paper and W, Z and An.

TABLE III
Differences in Magnitude Zero Point (N.G.C. 663)

Δ	Number of Stars	m_p	m_{pv}	CI
$A - W$	107	-0.85 ± 0.26	-0.54 ± 0.25	-0.31 ± 0.30
$A - Z$	38	-0.32 ± 0.22	-0.44 ± 0.25	$+0.12 \pm 0.28$
$A - An$	6	$+0.14 \pm 0.16$	$+0.02 \pm 0.33$	$+0.12 \pm 0.21$

Gushee's catalogue does not give the magnitudes but only the diameters without reduction to a reference scale. The diameters were therefore reduced to the A I pg magnitude scale and the agreement, represented by the mean deviation of $\pm 0^m.10$, is very satisfactory. The mean error of the A I scale is the same.

Wallenquist's results show a noticeable difference. Part of it is probably due to a different method of reduction, another part to the fog effect.* The reduction was done in two steps using an artificial scale between cluster stars and NPS stars, whereas in this paper the image diameters were directly reduced. The influence of the fog effect is indicated by the difference between long and short exposed magnitudes mentioned by Wallenquist himself. In the quoted fog paper it was shown that, on a plate with

* Alter, Barber, Edwards, *M.N.*, 100, 529, 1940.

two consecutive exposures, the stars of the second exposure appear too faint (Wallenquist's exposures are all in the order Pole-cluster), that this effect increases with increasing background luminosity (three of the four long exposed plates were taken on moonlight nights), that it increases with the exposure time (the reason for the difference between long and short exposure), and that it depends on the emulsion (the difference between m_p and m_{pv} , and thereby the influence on the CI, is explained by a special chemical treatment of the pv plates before exposing them). Thus, of the $-0^m.31$ difference in CI (Table III), $-0^m.16$ is due only to the differential fog effect between long and short exposure, and a part or all of the remaining $-0^m.15$ is probably due to the fog effect on the short exposure itself. (It may be noticed that in results here given the fog effect is removed.)

A further notable feature is the relation of the difference $A - W$ to the magnitude, shown in Table IV. The difference increases with the magnitude on the photographic plate while it has a maximum on the photovisual one. (This different behaviour is

TABLE IV
Differences in Magnitude Scale (N.G.C. 663)

m_p Range	Num- ber of Stars	$A - W$			Num- ber of Stars	$A - Z$		
		m_p	m_{pv}	CI		m_p	m_{pv}	CI
< 10	4	0.00 ± 0.08	$+0.02 \pm 0.20$	-0.02 ± 0.12	2	$+0.18 \pm 0.06$	$+0.19 \pm 0.23$	-0.01 ± 0.29
10-11	14	-0.54 ± 0.27	-0.44 ± 0.26	-0.10 ± 0.23	12	0.00 ± 0.20	-0.22 ± 0.23	$+0.22 \pm 0.24$
11-12	19	-0.90 ± 0.26	-0.78 ± 0.19	-0.12 ± 0.24	9	-0.45 ± 0.24	-0.64 ± 0.18	$+0.19 \pm 0.36$
12-13	48	-0.92 ± 0.27	-0.64 ± 0.26	-0.28 ± 0.35	13	-0.41 ± 0.26	-0.60 ± 0.33	$+0.19 \pm 0.27$
> 13	22	-1.03 ± 0.27	-0.30 ± 0.30	-0.73 ± 0.35	2	-1.26 ± 0.12	-0.11 ± 0.15	-1.15 ± 0.26

probably caused by the chemical treatment of the pv emulsion.) The change in $A - W$ with the magnitude suggests that the fog effect is dependent, in this case, on the magnitude. In the above-mentioned paper on fog effect it was found that the magnitude, expressed by image diameters, has no influence on the fog effect. In Wallenquist's investigation the magnitude was determined * by estimating the density as well as the size of the stellar images by a movable scale. The density, taking part in this determination, is, however, subjected to the fog effect as mentioned in the spectral part of the fog paper, but it is not known what is the final result of the combination of both effects. So it appears that a considerable part of the difference may be due to fog effect, but it seems rather difficult to explain the residue. A symmetrical arrangement of exposures, Pole-Cluster-Pole, could perhaps have shown a change in sky transparency, but the visibility was estimated during the exposures and found not to have changed within $0^m.1$. Even a change, when established, would not have produced the remarkable change of $A - W$ with the magnitudes.

The comparison with C. J. Anger's results confirms the influence of the fog effect. Only the 6 B.D. stars of my catalogue were available for comparison, the other stars in Anger's catalogue lying outside the cluster region. The differences have a positive sign against a negative sign with Wallenquist. This is also explained by the arrangement of the exposures. In Anger's case the order was, with one exception, cluster-Pole, so that the cluster stars appear brighter. The one exception was one plate among the 4 pv plates. By this inversion the fog effect worked in the other way and reduced

* *Medd. Upsala*, 32, 6, 1927.

the mean fog difference of the pv plates. Table III shows this result clearly: $A - An$ is much smaller for pv than for pg.

The CE of 0.50 noticed by Wallenquist is determined with Schalén's spectral classes and the latter's $Sp - CI$ table (*Medd. Upsala*, 49, 2). Reduced with Trumpler's spectral classes and King's table (used in this paper), the average CE for the 6 B.D. stars (members of W.'s and A.'s catalogues) is 0.72 against A.'s 0.75; a similar reduction brings Schalén's value from 0.58 to 0.77, and Anger's from 0.57 to 0.76, all of them yielding a good agreement.

The comparison with Zug gives a CI difference of +0.12 (Table III) which should also appear in the comparison of the CE. It does not, Zug's CE being +0.60 against my 0.62. The explanation is given by the method of reduction. Zug's CE is obtained by the use of his own $Sp - CI$ table. Reduced with King's table, a difference of +0.13 appears again. The differences $A - Z$ (Tables III and IV) are probably due to the different methods. Zug's magnitude determination is done in several steps and it is impossible to recognize at which step the differences appear first.

N.G.C. 659

R.A. = $1^h 37^m.4$, Dec. = $60^\circ 12'$ (1900.0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder, and investigated by Raab.

The present investigation was made on the same plates as in AI and the magnitudes are therefore subjected to the same reductions.

The star counting gave 59 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 3269 stars per sq. degree and 38 cluster stars within a cluster region of 5'.5 diameter. Fig. 1 gives the diagram of the star density per square degree within the cluster region and its surroundings.

Table V arranges the stars in two different ways. In the first part the absolute magnitudes were taken, as usual, from the main sequence. Here appear two ranges in the distance modulus which indicate two cluster formations. The first includes the stars between 38 and 3, the second between 6 and 25, the two being divided by the

TABLE VA

Co-ordinates, Magnitudes and Distance Moduli of 27 Stars (N.G.C. 659)

Zero point = HD 10425, unit = 1'.0, X = east, Y = north

Co-ordinates of the assumed cluster centre $X = +7.4$, $Y = +8.2$

Star	X	Y	m_p	m_{pv}	$m - M$	Star	X	Y	m_p	m_{pv}	$m - M$
17	+6.3	+8.1	12.56	11.33	4.3	58	+9.0	+9.8	13.39	13.40	12.3
38	7.4	9.1	11.65	10.80	5.7	1	4.9	8.1	13.39	13.46	12.8
11	5.9	5.9	11.85	11.02	6.0	46	7.9	6.8	12.87	12.97	13.3
51	8.4	8.5	12.30	11.40	6.2	42	7.8	6.6	13.49	13.64	13.9
19	6.4	9.1	12.10	11.32	6.5	44	7.8	8.9	13.16	13.35	14.4
55	8.6	7.7	11.45	10.81	6.5	13	6.2	6.5	13.67	13.90	15.5
37	7.3	8.1	11.05	10.62	7.3	25	6.7	9.6	12.86	13.14	15.5
4	5.2	9.6	12.53	11.93	8.0	30	6.9	8.6	13.63	13.90	16.2
9	5.6	7.7	12.10	11.32	8.4	59	9.6	7.9	13.38	13.71	16.7
45	7.8	7.6	10.31	10.18	8.8	23	6.7	6.0	13.58	14.05	17.5
56	8.7	9.0	11.29	11.15	9.6	10	5.8	6.6	13.47	13.99	17.7
3	5.0	7.3	12.97	12.57	10.1	2	4.9	9.1	13.78	14.53	18.5
6	5.3	8.4	13.16	13.13	11.9	54	+8.6	+6.5	13.57	14.54	18.5
41	+7.7	+5.6	13.52	13.39	12.0						

TABLE VB

Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>	Star	<i>m</i> - <i>M</i>
17	4.3	56	11.2	9	11.9	58	12.3	46	13.3	25	15.5	10	17.7
45	10.2	4	11.5	38	12.0	51	12.5	42	13.9	30	16.2	2	18.5
55	10.4	6	11.9	41	12.0	1	12.8	44	14.4	59	16.7	54	18.5
37	11.0	19	11.9	11	12.2	3	13.3	13	15.5	23	17.5		

big jump of *m* - *M* between 3 and 6. The mean distance modulus of the first group is 7.56, its distance 320 ± 30 parsecs, its diameter $3'.6 = 0.3$ parsecs, the values of the second group are, respectively, 13.51, 5000 ± 500 parsecs and $5'.5 = 9$ parsecs. The assumption that there are two clusters in exactly the same area is not probable and therefore not very satisfactory. This suspicion is, moreover, supported by the fact that in the nearer group no faint star appears and all stars are of later type, while in the farther group no bright one appears and all stars are of earlier type. Thus, the suggestion is offered that the group of later stars represents a group of giants. Using the giant values of *M* for them, the second part of Table V gives the redistribution of *m* - *M*. The range of the distance modulus fits quite well for both groups. This assumption seems to be more satisfactory than the first one, but it can be proved only by a spectral investigation of the red group. The mean value of *m* - *M* is 12.38 and, consequently,

$r = 3000 \pm 250$ parsecs, $d = 5'.5 = 4.8$ parsecs.

The results of the previous investigations were:

	Distance	Diameter
	Parsec.	Parsec.
Raab	...	6
Shapley	1200-1910	5 2.8
Trumpler	2340	5 3.4
Collinder	4170	5 6.0

The number of cluster stars, after correction for the background stars, is for the first assumption 20 stars for the nearer cluster and 19 for the farther one, and for the second assumption 33 stars, against 38 got statistically. The respective differences do not exceed the dispersion $\sigma = \pm 6$. The magnitude distribution is given in the following table:—

<i>m</i> _{pv}	11	12	13	> 13.5
Number of stars	7	1	6	19

N.G.C. 654

R.A. = 1^h 37^m.2, Dec. = +61° 23' (1900.0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder and investigated by Raab.

The present investigation was made on the same plates as in AI and the magnitudes are therefore subjected to the same reductions.

The star counting gave 60 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 1599 stars per square degree and in 44 cluster stars within a cluster region of 5'.5 diameter.

Table VI gives the distribution of *m* - *M*. The mean distance modulus of the

cluster group is 10.36 corresponding to $r = 1200 \pm 100$ parsecs. The low background density and the position of this cluster in the centre of a dark region suggest, as mentioned

TABLE VI

Co-ordinates, Magnitudes and Distance Moduli of 35 Stars (N.G.C. 654)

Zero point = star number 54 = HD 10494, unit = 1'.0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +1.4$, $Y = +2.4$

Star	X	Y	m_p	m_{pv}	$m - M$	Star	X	Y	m_p	m_{pv}	$m - M$
54	0.0	0.0	8.24	7.46	2.6	47	+2.8	+0.5	12.55	12.44	11.1
35	+0.6	+2.7	10.98	9.96	4.3	7	+2.6	+2.8	12.50	12.45	11.3
27	+1.3	+2.3	11.30	10.57	5.8	34	+0.7	+2.3	12.29	12.34	11.3
25	+1.4	+2.8	12.58	11.83	7.0	19	+1.7	+4.2	12.29	12.34	11.3
31	+1.1	+2.8	13.00	12.32	7.7	60	-1.7	+2.5	12.84	12.74	11.4
1	+3.1	+1.7	12.58	12.00	8.3	22	+1.5	+4.5	13.27	13.18	11.9
18	+1.8	+1.6	11.94	11.50	8.7	10	+2.3	+2.1	12.95	13.01	12.1
45	+3.9	+2.1	12.13	11.70	9.0	26	+1.3	+0.9	13.17	13.19	12.1
40	-0.1	+1.5	11.34	11.12	9.3	50	+2.4	0.0	11.39	11.54	12.3
56	-1.2	+0.8	12.01	11.70	9.4	28	+1.2	+3.6	13.69	13.74	12.7
23	+1.4	+4.2	12.32	12.01	9.7	37	+0.5	+2.1	13.64	13.78	13.8
2	+3.0	+2.6	11.71	11.51	9.8	55	-0.8	+0.7	13.26	13.74	17.2
5	+2.6	+3.2	11.79	11.56	9.8	6	+2.6	+1.7	13.58	14.12	17.8
44	-0.5	+2.3	12.72	12.40	10.0	39	+0.2	+2.9	13.63	14.17	18.0
57	-1.2	-0.4	12.15	11.99	10.4	36	+0.5	+1.2	12.87	14.21	18.2
16	+1.8	+2.8	11.59	11.64	10.6	58	-1.3	+1.3	13.37	14.16	18.2
14	+2.1	+2.6	12.55	12.39	10.8	24	+1.4	+1.8	13.71	14.48	18.8
41	-0.1	+3.0	12.95	12.74	11.0						

above, the assumption of the same absorption as for N.G.C. 663. As no spectral classification is available, a shift in the distance proportional to that of N.G.C. 663 was assumed. This brings as nearest approximation :

$$r = 2060 \pm 200 \text{ parsecs,} \quad d = 5'.5 = 3.3 \text{ parsecs.}$$

The results of the previous observations are

	Distance	Diameter	
	Parsecs	Parsecs	
Raab	...	6	...
Shapley	720-1150	5	1.7
Trumpler	1870	5.5	3.0
Collinder	3700	5	5.4

The number of cluster stars, after correction for the background stars, is 38 against 44 got statistically, the dispersion $\sigma = \pm 6$. The magnitude distribution is given in the following table:—

m_{pv}	11	12	13	> 13.5
Number of stars	1	16	4	17

N.G.C. 559

$$R.A. = 1^h 22^m.8, \quad \text{Dec.} = 62^\circ 47' (1900.0)$$

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder.

The present investigation was made on the same plates as in AI and the magnitudes are therefore subjected to the same reductions.

The star counting gave 61 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 2979 stars per square degree, and in 40 cluster stars within a cluster region of 4'.4 diameter.

Table VII gives the distribution of $m-M$. The mean distance modulus of the cluster group is 12.22 corresponding to

$r = 2770 \pm 250$ parsecs, $d = 4'.4 = 3.5$ parsecs.

TABLE VII

Co-ordinates, Magnitudes and Distance Moduli of 34 Stars (N.G.C. 559)

Zero point = star number 7 (brightest star), unit = 1'.0, X = east, Y = south
Co-ordinates of the assumed cluster centre X = +1.9, Y = +1.3

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
22	+1.3	+3.5	12.37	10.91	3.4	20	+1.3	-1.2	13.28	13.20	11.9
48	+2.8	+2.7	12.30	11.47	6.4	16	+1.0	+2.8	13.36	13.34	12.1
55	+3.8	+0.9	13.73	13.00	8.2	53	+3.3	+2.5	13.58	13.48	12.1
7	0.0	0.0	9.99	9.84	8.3	14	+0.9	+0.8	13.98	13.82	12.2
51	+3.2	+2.8	13.62	13.01	8.5	40	+2.4	-0.1	13.98	13.82	12.2
36	+2.3	-0.4	13.43	12.88	8.8	56	+3.8	+1.7	13.72	13.63	12.2
10	+0.4	+2.3	14.13	13.51	9.0	57	+4.0	+1.4	13.56	13.48	12.2
38	+2.3	+1.8	10.91	10.73	9.0	46	+2.8	+1.9	13.89	13.78	12.4
42	+2.6	+2.8	10.86	10.70	9.1	18	+1.2	-0.4	13.61	13.63	12.5
12	+0.6	+2.6	12.58	12.27	9.8	50	+3.1	+1.7	13.79	13.79	12.7
37	+2.3	-0.8	13.46	13.10	10.3	15	+0.9	+2.1	13.15	13.25	13.0
5	-0.3	+2.6	14.04	13.66	10.8	54	+3.6	+2.1	13.28	13.39	13.2
11	+0.5	-0.1	13.15	12.93	11.0	13	+0.6	+1.4	13.67	13.75	13.3
27	+1.7	+0.6	13.33	13.11	11.2	21	+1.3	+3.1	13.53	13.64	13.4
19	+1.2	+2.1	13.72	13.48	11.5	28	+1.7	+2.9	13.56	13.68	13.6
47	+2.8	-0.1	13.72	13.48	11.5	17	+1.1	+1.5	14.11	14.39	16.7
45	+2.7	+0.3	12.98	12.90	11.6	26	+1.5	+1.0	13.82	14.15	17.2

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs		Parsecs
Shapley	1740-2750	7	5.6
Trumpler	2290	6	4.0
Collinder	1600	7.5	3.5

The number of cluster stars (no correction is necessary because no background stars are within the modulus range) is 38 against 40 got statistically; the dispersion is $\sigma = \pm 6$. The magnitude distribution is given in the following table:—

m_{pv}	13	> 13.5
Number of stars	7	31

Tr. I

R.A. = 1^h 29^m.0, Dec. = 60° 46' (1900.0)

This cluster has first been mentioned in Trumpler's catalogue as Anonymous 1 and is also registered in Collinder's catalogue. The occurrence of this group of stars, however, is mentioned in 1893 by Roberts (*M.N.*, 53, 332) as "very remarkable."

The present investigation was made on the same plates as in AI and the magnitudes are therefore subjected to the same reductions.

The star counting gave 63 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 3253 stars per square degree, and in 31 cluster stars within a cluster region of 5'·3 diameter.

Table VIII gives the distribution of $m-M$. The mean distance modulus of the cluster group is 11·67, corresponding to

$r=2150 \pm 200$ parsecs, $d=5' \cdot 3=3 \cdot 3$ parsecs.

TABLE VIII
Co-ordinates, Magnitudes and Distance Moduli of 28 Stars (Tr. 1)

Zero point = B.D. 60° 403, unit = 1'·0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +7 \cdot 3$, $Y = +0 \cdot 4$

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
31	+6·7	-0·6	11·04	10·60	7·3	37	+6·2	+1·4	13·53	13·42	12·0
38	+6·2	-0·4	11·25	10·80	7·4	43	+9·7	+1·3	13·45	13·43	12·2
1	+9·3	+0·2	12·74	12·18	8·1	11	+8·2	+1·9	13·17	13·24	12·4
28	+7·0	+0·4	10·06	9·89	8·3	22	+7·3	+1·9	12·99	13·09	12·8
51	+7·6	+2·8	10·38	10·30	9·0	36	+6·4	+0·9	13·26	13·44	14·2
20	+7·4	-0·1	10·45	10·48	9·5	50	+7·7	-1·9	12·61	12·87	14·9
24	+7·2	-0·9	11·68	11·57	10·2	3	+8·9	+1·8	13·13	13·48	16·7
25	+7·2	+0·2	10·38	10·48	10·2	34	+6·4	+1·1	12·98	13·49	17·5
47	+8·6	+2·6	12·23	12·06	10·5	35	+6·4	-1·5	13·19	13·65	17·6
30	+6·8	+0·5	10·73	10·84	10·6	33	+6·4	+1·7	13·52	14·13	18·1
10	+8·2	-0·2	12·65	12·45	10·7	62	+4·7	+1·3	13·52	14·06	18·1
4	+8·8	+0·3	13·03	12·79	10·8	6	+8·7	+0·8	13·56	14·18	18·2
55	+6·3	-2·0	12·95	12·81	11·3	12	+8·1	-0·8	13·47	14·21	18·2
5	+8·7	-0·8	12·88	12·81	11·5	60	+4·7	+0·2	13·19	13·54	18·7

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs	Parsecs	
Trumpler	2290	4·5	3·0
Collinder	3330	4·0	3·9

The number of cluster stars, after correction for the background stars, is 29 against 31 got statistically; the dispersion $\sigma = \pm 5$. The magnitude distribution is given in the following table:—

m_{pv}	10	11	12	13	>13·5
Number of stars	2	1	1	8	17

Appendix to N.G.C. 581

The co-ordinates of the assumed cluster centre are +1·8 and +2·5, respectively.

A comparison of the results of AI with Zug's paper on this cluster is not satisfactory because the number of comparison stars is too small for the statistical procedure. Among the 12 stars with spectral classes, noted by Zug, one is off the cluster region, the image of one is covered on Sidmouth plates by a brighter star, 3 more are excluded by Zug as not being cluster members. Still, the remaining 7 stars indicate with an average CE of -0^m·03 that there is no absorption in this region.

Summary

The cluster investigation of the present paper extends, in available cases, the method described in an earlier paper by the use of spectral classification to determine a possible colour excess. A comparison of colour excesses, published by Zug, with background star densities, published by Raab, for the same 15 clusters seems to indicate a relation between these two quantities. Irregularity in the range of the distance modulus ($m - M$) of cluster stars indicates the presence of absorption.

Five clusters in a small Cassiopeia region are investigated. N.G.C. 663 is shown to be obscured, having a remarkable colour excess. A comparison with earlier investigations of this cluster is given, and the differences, as far as possible, explained. N.G.C. 654 is supposed to be covered by the same obscuring cloud as the foregoing cluster, and a suitable correction is applied. N.G.C. 659 shows a notable distribution of distance moduli, thus giving two possibilities of explanation: either the clustering in the area is the result of a superimposition of two different widely separated clusters, or, more probably, it represents one cluster with a group of giants. Finally, the results are given of N.G.C. 559 and a cluster, first mentioned in Trumpler's catalogue, as Tr. 1.

Tables showing co-ordinates, photographic and photovisual magnitudes of all measurable stars within the above cluster regions have been prepared, but owing to shortage of paper it is regretted that these cannot be printed. Copies can be supplied on request to those specially interested.

*Norman Lockyer Observatory,
Sidmouth :
1940 November 29.*

OBSERVATION AT SYDNEY OBSERVATORY OF THE TRANSIT OF MERCURY, 1940 NOVEMBER 11-12

(Communicated by the Government Astronomer)

(Received 1941 February 6)

The following observations of the contact times (G.M.T.) of the transit of Mercury on 1940 November 11-12 were made by H. W. Wood and E. E. Adderley. At the time of ingress there was a good deal of hazy cloud, and the limb of the Sun and outline of the planet were both a little diffuse and boiling somewhat. At egress, conditions were quite clear but still unsteady. Both observers noted the "black drop" at the second and third contacts, being less marked at the third. Nothing in the nature of a white dot or halo round the planet, such as have been reported on past occasions, was observed.

H. W. Wood observed the ingress through a $7\frac{1}{2}$ -inch refractor, using a power of 140, the times being taken by eye and ear. Mercury was first seen at $20^h 50^m 00^s$, when it was almost half on the Sun's disc, and second contact was noted at $20^h 51^m 15^s$, uncertain by probably less than five seconds. The third and fourth contacts were observed through the $11\frac{1}{2}$ -inch refractor, using a polarizing eyepiece with an ocular of power 96, the times being taken again by eye and ear. The thread between the Sun's limb and the planet appeared to be broken momentarily at $1^h 51^m 05^s$, and was made and broken several times. Contact on this account was somewhat uncertain, but was definitely complete at $1^h 51^m 21^s$. Fourth contact was observed at $1^h 52^m 58^s$.

E. E. Adderley used the 6-inch refractor fitted with a Herschel solar diagonal and a Huygenian ocular of power 106, the time for each contact being recorded on a chronograph by means of a tapping key. Second contact was noted at $20^h 51^m 20^s$ (believed within five seconds of the correct value). The third contact was noted at $1^h 51^m 22^s$, and was definitely complete two seconds after this.

In the following table the formulæ of the *Nautical Almanac*, 1940, page 512, are used to reduce the times to the centre of the Earth.

Contact	Observer	Observed Contact	Correction to Centre	Geocentric Time
		h m s	s	h m s
II	H.W.	20 51 15	-35	20 50 40
	E.A.	20 51 20	-35	20 50 45
III	H.W.	1 51 05	+11	1 51 16
	E.A.	1 51 22	+11	1 51 33
IV	H.W.	1 52 58	+11	1 53 09

OBSERVATION OF THE TRANSIT OF MERCURY ON

1940 NOVEMBER 11-12

D. W. N. Stibbs

(Communicated by the Director, Commonwealth Solar Observatory, Canberra)

(Received 1941 March 17)

The Transit of Mercury of 1940 November 11-12 was observed on Mount Stromlo with the Sun Tower Telescope, which contains a 12-inch aperture photographic doublet lens of focal length 43 feet.

For the purpose of photographing the transit, the lens aperture was reduced to 5 inches and an exposure of $1/10000$ sec. given to the photographic plates (Kodak Nepera) on each of which was recorded the position of the east and west points on the solar disc. Times of exposure of the plates were recorded by means of a chronograph which also recorded the second beats of the Observatory Time Service, the error of which was standardised against time-signals received from NPG Mare Island.

Throughout the transit seeing conditions were bad, an overcast sky preventing any exposures being made until about $1^h 40^m$ after ingress. As a number of the photographs were taken through cloud, the images on some of the plates were somewhat diffuse, but on some of the better plates taken towards the egress it was possible to measure the planet's diameter.

Using a travelling microscope the position of the centre of the planet was measured relative to tangents to the solar limb. The results are expressed as rectangular co-ordinates of the centre of Mercury relative to the centre of the Sun, on the scale Sun's radius = 1.0000. The diameter of Mercury measured on three plates was 0.53 mm. Since the Sun's diameter was 123.48 mm. and the *Nautical Almanac* gives $971''.5$ for the semi-diameter at the time, the semi-diameter of Mercury from the plates is $4''.2$ (cf. *N.A.*, $4''.9$).

Of the four contacts only the exterior contact at egress was observed, but the times of all four contacts have been obtained by extrapolation from the results of the photographic observations. The observed mean position angles of ingress and egress are $92^\circ.1$ and $316^\circ.4$ respectively.

For interest, the times of the four contacts obtained by extrapolation have been reduced to Geocentric Times. Predicted Geocentric Times for a $4''.2$ semi-diameter Mercury have been obtained from information given in the *Almanac*, but the figures given are not strictly intended for the comparison of observation with theory.

Universal Time				Co-ordinates *		Universal Time				Co-ordinates *	
d	h	m	s	x	y	d	h	m	s	x	y
Nov. 11	22	30	38.2	-0.44056	+0.22076	Nov. 12	01	28	33.3	+0.55620	+0.66505
	12	00	00	+0.07839	0.44234		01	37	13.1	0.60560	0.69015
		00	05	0.08925	0.46080		01	40	13.6	0.61791	0.69258
		00	57	0.37803	0.59232		01	47	06.9	0.65727	0.70845
		01	18	0.49595	0.64529						

* Note.—The +x axis crosses solar limb at the North Point on the solar disc.

	Ingress		Egress	
	Exterior Cont.	Interior Cont.	Interior Cont.	Exterior Cont.
	h m	h m	h m	h m
Observed	...	...	...	01 53.08
By Extrapolation	20 49.93	20 51.34	01 51.68	01 53.08
Geocentric Times	20 49.35	20 50.75	01 51.86	01 53.26
Predicted	20 49.42	20 50.94	01 52.27	01 53.79

Commonwealth Solar Observatory,
Mount Stromlo, Canberra, A.C.T.:
1940 December.

STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE CAPE OBSERVATORY (TWELFTH LIST)

(Communicated by H.M. Astronomer)

(Received 1940 October 3)

This list is presented in the same form as previous lists.

The number of plates taken in recent years had considerably exceeded the rate of measurement, and complete series for a number of stars are awaiting measurement. This has the advantage that a whole series is measured about the same time by the same measurer. The plates for a star now generally extend over seven epochs at least and exceed 30 in number.

The list contains a number of binaries or close pairs of stars with common proper motion, viz. 1008-9, 1014-15, 1028, 1033-34, 1040-41, 1058-59, 1079-80. No. 1068 is a close double.

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900.0	Unit 0".001				
					Proper Motion		P.E. Unit Parallax		P.E.
					R.A.	Dec.	Weight		
1001	P.G.C. 291 40 Ceti	6.83	F8	^{h m s} 1 11 52	+267	-129	31	+43	± 9
		7.33		-2° 48'.2	+272				± 8
1002	Ci. 18, 190 -1° 184	8.8	...	1 20 05	-171	-292	26	+38	± 9
		10.5		-1° 34'.4	-192				± 7
1003	F.S. 561 C.A.Z. -49° 25	...	...	2 02 19	+194	-72	25	+26	± 8
		13.0		-48° 39'.4	+181				± 8
1004	F.S. 643 C.A.Z. -49° 13	...	...	2 24 49	-81	-228	28	+13	± 8
		12.7		-48° 24'.8	-105				± 7
1005	F.S. 676 C.A.Z. -49° 132	...	Go	2 32 19	+197	-111	27	+40	± 8
		12.5		-48° 06'.5	+154				± 6
1006	G.C. 3431 -44° 863	8.0	Go	2 47 02	-17	-275	26	+59	± 8
		8.2		-44° 29'.2	-58				± 5
1007	G.C. 3692 -5° 570	9.9	G	2 59 56	-6	-44	22	+14	± 7
		10.4		-5° 31'.7	-26				± 6
1008	Z.C. 1624 -50° 1015 p.	...	...	3 20 10	+216	+295	20	+48	± 6
		12.1		-50° 21'.4	+220				± 3
1009	Z.C. 1625 -50° 1015	8.6	Ko	3 20 11	+228	+268	28	+48	± 8
		9.4		-50° 21'.2	+232				± 4
1010	κ Reticuli P.G.C. 812	4.80	F5	3 27 38	+366	+369	24	+49	± 6
		5.22		-63° 17'.4	+370				± 3
1011	F.S. 1102 -41° 1163	9.0	Go	3 47 25	-115	-195	25	+19	± 8
		10.3		-41° 44'.9	-120				± 6
1012	δ Horologii P.G.C. 966	4.85	Fo	4 07 29	+192	+65	19	+14	± 6
		5.13		-42° 15'.3	+173				± 5

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900.0	Unit 0".001				
					Proper Motion		P.E. Unit Weight	Parallax	P.E.
					R.A.	Dec.			
1013	<i>a</i> Horologii P.G.C. 985	3.83 4.83	Ko	^{h m s} 4 10 41 -42° 32'.5	+ 41 + 12	- 215	24	+ 27	± 7 ± 5
1014	G.C. 5365 -57° 659 <i>p</i> .	7.4 ...	...	4 22 15 -57° 17'.9	- 105 - 144	- 54	28	+ 44	± 7 ± 6
1015	G.C. 5366 -57° 659	6.54 7.4	Go	4 22 16 -57° 17'.8	- 102 - 156	- 79	20	+ 33	± 5 ± 4
1016	G.C. 5804 -47° 1497	8.5 8.9	Go	4 41 51 -47° 35'.6	+ 253 + 245	- 38	23	+ 26	± 7 ± 6
1017	F.S. 1516 -43° 1604	... 11.2	F5	4 51 45 -43° 22'.6	+ 179 + 185	- 21	20	+ 32	± 7 ± 6
1018	Z.C. 2560 -51° 1309	8.3 9.2	G5	4 53 48 -51° 12'.0	+ 174 + 186	- 133	25	+ 29	± 7 ± 6
1019	F.S. 1558 -50° 1587	... 11.4	...	4 56 53 -50° 26'.2	- 10 - 7	- 248	14	- 4	± 4 ± 4
1020	G.C. 6363 -44° 1905	8.6 10.0	G5	5 07 44 -44° 42'.0	+ 95 + 89	+ 194	37	+ 37	± 12 ± 9
1021	F.S. 1663 -41° 1766	... 11.2	Ko	5 10 31 -41° 00'.6	+ 347 + 339	- 203	32	+ 26	± 11 ± 7
1022	F.S. 1698 -48° 1741	... 10.7	F5	5 15 41 -48° 58'.1	+ 182 + 196	- 242	28	+ 21	± 8 ± 6
1023	Ci. 20333 Wolf 1450	... 13.5	...	5 24 55 -3° 30'.4	- 329 - 309	- 480	37	+ 57	± 11 ± 6
1024	H.D. 37656 -62° 494	9.4 10.6	K5	5 34 59 -62° 52'.1	- 90 - 87	+ 250	30	+ 39	± 8 ± 7
1025	F.S. 1817 -44° 2153	... 11.3	...	5 35 22 -44° 14'.2	+ 94 + 110	+ 143	29	+ 2	± 9 ± 6
1026	F.S. 1891 C.A.Z. -46° 86	... 13.5	...	5 45 57 -45° 57'.9	- 177 - 134	+ 230	28	+ 16	± 8 ± 6
1027	F.S. 1950 -45° 2231	... 11.3	...	5 53 06 -45° 12'.5	+ 73 + 68	+ 208	44	- 9	± 12 ± 9
1028	G.C. 7909 <i>p</i> . -1° 1146	9.0	A	6 07 33 -1° 16'.5	- 27 - 15	- 8	33	+ 14	± 10 ± 6
	G.C. 7909 <i>f</i> . -1° 1146	...	...	...	...	...	35	+ 34	± 10 ± 6
1029	G.C. 8298 -0° 1287	5.85 6.35	F8	6 20 10 -0° 53'.0	+ 223 + 264	- 224	33	+ 24	± 10 ± 6
1030	F.S. 2260 -49° 2277	... 12.7	...	6 26 54 -49° 53'.1	+ 219 + 228	- 117	31	- 3	± 9 ± 6
1031	F.S. 2384 -48° 2445	... 10.6	F8	6 38 46 -48° 07'.8	+ 67 + 73	+ 215	19	+ 13	± 6 ± 4

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900-0	Unit 0".001				
					Proper Motion		P.E. Unit Weight	Parallax	P.E.
					R.A.	Dec.			
1032	F.S. 2408 -41° 2558	... 11.6	...	^{h m s} 6 41 02 -41° 34'.5	+214 +216	+114	19	+16	± 6 ± 4
1033	F.S. 2513 -44° 2973	... 10.6	G0	6 48 38 -44° 10'.7	+121 +127	+146	22	+18	± 6 ± 4
1034	Z.C. 3959 -44° 2974	7.8 8.8	K0	6 48 40 -44° 11'.4	+124 +125	+152	18	+19	± 5 ± 3
1035	F.S. 2578 -46° 2774	... 10.7	G5	6 53 15 -46° 56'.2	-176 -167	+223	37	+20	± 10 ± 8
1036	G.C. 9662 -48° 2814	8.0 8.0	G0	7 12 45 -48° 03'.4	-152 -145	+167	28	+12	± 8 ± 5
1037	-4° 1915 A.R. Monoc.	9 Var.	K0	7 15 52 -5° 04'.4	... + 18	...	23	- 4	± 7 ± 6
1038	G.C. 9824 -1° 1677	9.0 10.0	K0	7 17 09 -1° 40'.7	-346 -358	+238	26	- 1	± 8 ± 5
1039	G.C. 10112 -49° 2901	8.5 9.2	G5	7 29 39 -49° 38'.4	+ 8 + 13	+237	24	+23	± 6 ± 4
1040	G.C. 10451 -50° 2947	6.75 7.1	F5	7 41 32 -50° 13'.1	-118 - 80	+135	21	+33	± 5 ± 4
1041	G.C. 10452 -50° 2948	7.9 8.1	G0	7 41 36 -50° 13'.7	-112 - 80	+131	26	+16	± 7 ± 5
1042	F.S. 3480 -49° 3115	... 10.8	K2	7 48 20 -50° 03'.2	-136 -151	+152	22	+44	± 6 ± 5
1043	G.C. 10694 -1° 1883	7.5 8.3	G5	7 49 31 -1° 08'.9	-265 -241	- 58	24	+49	± 6 ± 4
1044	P.G.C. 2099 11 Puppis	4.35 4.85	F8	7 52 34 -22° 36'.8	- 40 - 45	+ 8	25	+11	± 7 ± 5
1045	-34° 4160 Alb. 2488	8.3 8.3	G5	7 54 54 -34° 40'.7	-366 -330	+171	25	+38	± 7 ± 6
1046	Z.C. 5251 -45° 3697	7.7 7.8	G0	7 59 21 -46° 03'.7	-125 -123	+218	19	+20	± 5 ± 4
1047	F.S. 4104 -51° 2934	... 11.5	...	8 15 20 -51° 39'.2	+132 +110	-126	26	+30	± 8 ± 7
1048	F.S. 4150 C.A.Z. -46° 539	... 12.5	...	8 17 38 -45° 48'.8	- 80 - 87	+186	16	+24	± 4 ± 3
1049	F.S. 4451 -49° 3617	... 11.5	...	8 28 57 -49° 51'.3	-200 -192	+211	27	+51	± 10 ± 9
1050	F.S. 4465 -51° 3073	... 11.0	...	8 29 45 -51° 10'.9	-113 - 94	+225	28	-26	± 8 ± 6
1051	-51° 3074	... 10.8	...	8 29 46 -51° 10'.0	... + 18	...	36	- 6	± 10 ± 8

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900.0	Unit 0".001				
					Proper Motion		P.E. Unit Parallax		P.E.
					R.A.	Dec.	Weight		
1052	F.S. 4772 -49° 3760	... 12.0	...	^{h m s} 8 40 08 -49° 46'.3	- 91 - 92	+224	22	+19	± 6 ± 5
1053	δ Velorum P.G.C. 2356	2.01 2.01	Ao	8 41 57 -54° 20'.5	+ 24 + 28	- 93	24	+49	± 7 ± 4
1054	G.C. 12153 -41° 4507	6.48 7.2	Go	8 44 02 -41° 21'.7	- 29 + 18	-220	28	+25	± 8 ± 6
1055	F.S. 5142 -51° 3317	... 11.8	...	8 50 36 -51° 19'.6	- 8 - 11	-344	18	+24	± 5 ± 4
1056	F.S. 5268 -51° 3382	9.4 10.3	Ko	8 54 19 -51° 17'.3	-242 -246	+153	32	+35	± 9 ± 6
1057	G.C. 12608 -6° 2825	8.4 9.0	Go	9 02 56 -6° 44'.0	+ 86 + 95	- 84	26	+ 5	± 9 ± 8
1058	F.S. 5662 -44° 5200 <i>p.</i>	... 11.2	...	9 07 15 -44° 54'.0	-198 -183	+ 36	31	+52	± 9 ± 8
1059	F.S. 5663 -44° 5200 <i>f.</i>	... 10.3	Ko	9 07 16 -44° 54'.0	-182 -190	+ 28	26	+35	± 8 ± 7
1060	F.S. 7007 C.A.Z. -50° 126	... 12.3	...	9 55 09 -49° 31'.5	-144 -164	+ 34	26	+43	± 8 ± 5
1061	F.S. 7030 -50° 4715	... 12.1	...	9 55 39 -50° 07'.4	-186 -188	+ 59	17	+40	± 6 ± 6
1062	Ci. 20, 569 Ross 445	... 12.0	...	10 10 09 -9° 09'.8	-520 -540	- 60	24	+26	±10 ± 6
1063	Hyderabad A.C. -17° 11 ^h 0 ^m , 31766	... 12.5	...	10 57 01 -17° 25'.5	... + 5	...	19	+53	± 5 ± 4
1064	G.C. 15546 -4° 3049	7.34 8.34	Ko	11 13 11 -4° 31'.0	+791 +804	-148	24	+41	± 8 ± 4
1065	λ Crateris P.G.C. 2997	5.15 5.49	F2	11 18 24 -18° 13'.8	-312 -298	- 40	36	+30	±10 ± 7
1066	P.G.C. 3049 Z.C. 9642	5.57 7.4	Ko	11 30 10 -48° 35'.3	-179 -164	+161	21	+49	± 6 ± 5
1067	-11° 3135	9.3 9.7	F5	11 31 37 -11° 49'.0	- 13 - 9	+ 45	28	+ 4	± 9 ± 7
1068	A.D.S. 8239 -11° 3136	9.3 10.1	G5	11 31 46 -11° 47'.8	+ 22 + 19	- 43	20	+31	± 7 ± 6
1069	F.S. 9444 -50° 6462	... 11.7	G5	11 49 26 -50° 50'.2	-253 -247	+249	18	+37	± 5 ± 5
1070	F.S. 9496 -50° 6525	... 11.3	G5	11 52 56 -50° 56'.7	-238 -221	+ 37	25	+18	± 8 ± 8
1071	Ci. 20, 684 Wolf 406	... 12.3	...	12 03 22 +0° 04'.6	-960 -937	- 80	27	+73	± 8 ± 4

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900.0	Unit 0".001				
					Proper Motion		P.E. Unit Parallax	P.E.	
					R.A.	Dec.			
1072	F.S. 10155 -51° 6859	... 11.6	...	^{h m s} 12 32 30 -51° 27'.1	-1038 -1023	+ 32	28	+115	± 9 ± 7
1073	...	... 13.2	...	13 18 29 -13° 31'.2	... + 9	...	41	+12	±12 ±10
1074	Wolf 482 Ross 985	... 13.2	...	13 18 30 -13° 30'.7	-480 -656	-370	35	+47	±10 ± 9
1075	F.S. 11732 -50° 8237	... 11.1	G5	13 58 38 -50° 52'.5	-159 -165	+ 17	28	+17	± 7 ± 7
1076	U.O.C. 48	... 13.2	...	14 24 01 -53° 38'.9	-270 -193	-196	26	+51	± 8 ± 7
1077	F.S. 12626 -43° 9510	... 11.4	K7	14 52 57 -43° 41'.2	-274 -260	-269	30	+54	± 7 ± 7
1078	Ci. 20, 911 -67° 2775	9.6 ...	...	15 02 02 -67° 37'.1	-340 -304	-320	21	+45	± 6 ± 4
1079	G.C. 20783 -8° 3981	6.80 7.80	Ko	15 22 45 -8° 59'.4	+ 67 + 37	-360	23	+59	± 6 ± 5
1080	G.C. 20785 -8° 3983	8.1 9.1	Ko	15 22 47 -9° 00'.0	+ 92 + 95	-369	28	+82	± 7 ± 7
1081	Ci. 20, 944 Ross 804	... 12.8	...	15 39 56 -13° 30'.0	-200 -111	-550	37	- 6	±12 ± 9
1082	Ci. 20, 970 -52° 9317	... 11.3	...	16 04 09 -52° 38'.5	-190 -247	-520	26	+44	± 7 ± 5
1083	G.C. 22206 -63° 3955	7.6 8.2	Go	16 23 55 -63° 37'.3	-439 -438	-194	25	+64	± 7 ± 7
1084	G.C. 22196 -38° 11019	7.49 7.6	G5	16 24 47 -38° 47'.2	-415 -438	-330	34	+47	± 9 ± 8
1085	Z.C. 15017 -40° 10550	8.8 9.7	Ko	16 30 45 -40° 40'.4	-372 -375	-376	20	+33	± 6 ± 6
1086	Ci. 20, 1008 -30° 13458	9.7 11.5	...	16 40 24 -30° 26'.4	-140 -132	+420	29	+39	± 9 ± 7
1087	Z.C. 15421 -51° 10564	9.0 9.0	Go	16 45 21 -51° 32'.8	-183 -147	- 88	25	+10	± 7 ± 7
1088	Ci. 20, 1108 Ross 154	... 12.2	...	18 43 37 -23° 56'.5	+710 +641	-180	17	+336	± 4 ± 4
1089	γ Cor. Aust. P.G.C. 4851	5.01 5.51	F8	18 59 40 -37° 12'.4	+ 91 + 74	-276	35	+56	±10 ± 9
1090	G.C. 29030 -41° 14250	8.8 9.5	Go	20 43 50 -40° 58'.0	- 68 - 70	-454	39	+25	±12 ± 8
1091	Ci. 20, 1236 Wolf 891	... 11.8	...	20 48 48 -3° 08'.2	-644 -642	-370	36	+15	±10 ± 7

No.	Name and B.D. No.	Mag. Vis. Phot.	Spec. Type	R.A. Dec. 1900.0	Unit 0".001				
					Proper Motion		P.E. Unit Parallax	P.E.	
					R.A.	Dec.			
				h m s			Weight		
1092	C.A.Z. -42° 338	...	...	20 54 16	...	...	23	- 5	± 7
		12		-42° 26'.3	+ 36				± 6
1093	F.S. 19343	...	...	20 54 20	+434	+ 28	25	+41	± 8
	C.A.Z. -42° 344	12.3		-42° 25'.1	+413				± 6
1094	U.O.C. 53	...	...	22 31 28	+874	-179	27	+94	± 8
		12.8		-65° 53'.8	+800				± 7
1095	Z.C. 20271	8.7	G5	22 59 53	+280	-149	14	+ 6	± 4
	-48° 14408	9.5		-48° 24'.5	+268				± 4
1096	Z.C. 20304	8.8	K0	23 03 13	+137	-188	23	+14	± 7
	-43° 15275	9.6		-42° 59'.3	+131				± 7
1097	Z.C. 20398	9.1	G5	23 11 32	+114	-212	20	+24	± 5
	-46° 14601	9.9		-46° 13'.1	+ 92				± 5
1098	F.S. 20301	11.8	G0	23 16 06	+215	-279	26	+26	± 7
	-50° 13953	11.7		-50° 01'.7	+216				± 7
1099	F.S. 20395	9.9	K2	23 30 31	-225	-336	25	+ 4	± 7
	-46° 14707	10.9		-46° 42'.5	-262				± 7
1100	F.S. 20522	...	K7	23 52 03	+313	- 34	35	+28	±10
	-41° 15360	11.2		-41° 40'.0	+272				± 8

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN 1940

(Communicated by H.M. Astronomer)

(Received 1941 February 11)

The occultations were reduced by the method and with the constants given in the *Nautical Almanac*.

Observers:—Dr. J. Jackson.	J.	Mr. A. Menzies.	A.M.
Dr. R. H. Stoy.	S.	Mr. J. B. Turner.	J.T.
Mr. J. H. Peirce.	H.P.	Mr. W. P. Hirst.	H.
Mr. G. W. Reed.	G.R.		

Co-ordinates of instruments used:

Instrument	Longitude	Latitude	$\rho \cos \phi'$	$\rho \sin \phi'$
	^h ^m ^s	[°] ['] ["]		
10-inch equatorial	1 13 54.70 E.	-33 56 07.8	0.83054	-0.55509
18- " "	54.57	06.8	4	9
8- " "	54.57	06.8	4	9
7- " "	54.64	07.0	4	9
6- " "	54.69	07.1	4	9
Heliometer	54.65	01.4	6	7

A correction of $+1''.5$ was added to the Moon's mean longitude as given in the *Nautical Almanac*.

An analysis of the equations of condition gives the corrections to the orbital longitude ΔL and latitude ΔB :

Mean Epoch		ΔL	Wt.	ΔB	Wt.		
1940.55		$+1''.04$	50.5	$-0''.78$	26.2		
No.	Date	Star	Mag.	Phase	Inst.	Obs.	U.T.
	1940						^h ^m ^s
1	Jan. 21	B.D. $+19^\circ 898$	6.8	D.D.	10-inch	S.	22 20 07.8
2	" 27	43 Leonis	6.3	R.D.	10- "	S.	0 53 56.4
3	" 27	75 Leonis	5.4	"	10- "	S.	22 42 50.6
4	Feb. 15	B.D. $+15^\circ 430$	6.6	D.D.	10- "	S.	19 05 18.3
5	" 15	"	"	"	7- "	H.	19 05 20.3
6	" 19	B.D. $+18^\circ 1338$	6.8	"	10- "	J.	18 35 56.5
7	" 19	"	"	"	6- "	S.	18 35 57.3
8	" 19	"	"	"	7- "	H.	18 35 57.4
9	Mar. 1	24 Scorpii	5.0	R.D.	10- "	S.	2 24 54.4
10	" 18	B.D. $+17^\circ 1561$	7.4	D.D.	10- "	J.	19 45 33.0
11	" 18	"	"	"	7- "	H.	19 45 33.3
12	" 19	B.D. $+15^\circ 1805$	7.5	"	7- "	H.	20 13 40.8
13	" 20	B.D. $+12^\circ 2009$	6.3	"	10- "	H.P.	18 33 30.9
14	" 20	"	"	"	6- "	G.R.	18 33 31.0
15	Apr. 17	B.D. $+9^\circ 2269$	"	"	10- "	J.	21 56 50.1
16	" 17	"	"	"	6- "	S.	21 56 50.1
17	" 17	"	"	"	7- "	H.	21 56 50.2
18	" 17	π Leonis	4.9	"	10- "	J.	23 08 54.1
19	" 17	"	"	"	6- "	S.	23 08 54.2
20	" 17	"	"	"	7- "	H.	23 08 54.2

No.	Date 1940	Star	Mag.	Phase	Inst.	Obs.	U.T.		
							h	m	s
21	May 11	B.D. +18° 1349	6.2	D.D.	7-inch.	H.	17	35	56.5
22	" 12	B.D. +16° 1551	7.4	"	6- "	G.R.	18	46	19.6
23	" 12	"	"	"	7- "	H.	18	46	19.8
24	" 14	o Leonis	3.8	"	7- "	H.	21	18	28.9
25	" 14	"	"	"	6- "	S.	21	18	28.9
26	June 17	θ Libræ	4.3	"	7- "	H.	19	12	03.3
27	" 21	55 Sagittarii	5.1	R.D.	7- "	H.	19	41	59.2
28	" 26	21 Piscium	5.8	"	7- "	S.	23	09	11.2
29	July 12	B.D. -8° 3639	6.2	D.D.	7- "	H.	20	47	22.3
30	" 15	24 Scorpii	5.0	"	7- "	H.	23	12	20.1
31	Aug. 9	2 Libræ	6.3	"	7- "	J.	17	06	15.6
32	" 9	"	"	"	18- "	J.T.	17	06	15.7
33	" 9	"	"	"	6- "	A.M.	17	06	15.9
34	" 9	B.D. -11° 3736	6.5	"	18- "	J.T.	17	56	47.6
35	" 9	"	"	"	7- "	J.	17	56	48.0
36	" 9	"	"	"	6- "	A.M.	17	56	48.1
37	" 10	o ³ Libræ	6.7	"	7- "	S.	19	04	37.8
38	" 10	"	"	"	6- "	A.M.	19	04	37.9
39	" 10	B.D. -14° 4208	7.4	"	7- "	S.	21	32	17.0
40	" 10	"	"	"	6- "	A.M.	21	32	17.1
41	" 11	B.D. -16° 4280	6.7	"	18- "	J.	20	36	54.1
42	" 12	B.D. -17° 4759	7.4	"	7- "	H.	18	51	38.1
43	" 15	61 Sagittarii	5.0	"	Heliom.	S.	16	44	14.4
44	" 15	"	"	"	7-inch	H.	16	44	15.0
45	" 15	"	"	"	6- "	A.M.	16	44	15.0
46	" 26	B.D. +16° 560	6.1	R.D.	6- "	A.M.	2	28	28.0
47	Sept. 8	B.D. -17° 4700	7.2	D.D.	6- "	S.	20	27	16.1
48	" 8	"	"	"	7- "	H.	20	27	16.1
49	" 8	B.D. -17° 4717	6.1	"	6- "	S.	22	01	58.8
50	" 8	"	"	"	7- "	H.	22	01	58.9
51	" 10	B.D. -17° 5347	6.8	"	18- "	J.	17	20	54.8
52	" 10	"	"	"	7- "	H.	17	20	55.1
53	" 10	"	"	"	6- "	A.M.	17	20	55.2
54	Oct. 6	B.D. -17° 4903	7.5	"	7- "	H.	20	15	40.0
55	" 6	"	"	"	6- "	A.M.	20	15	40.1
56	" 9	B.D. -14° 5732	6.7	"	Heliom.	S.	17	47	46.9
57	" 9	"	"	"	18-inch	J.T.	17	47	47.0
58	" 9	"	"	"	6- "	A.M.	17	47	47.2
59	" 9	"	"	"	7- "	H.	17	47	47.3
60	" 9	B.D. -14° 5734	7.0	"	8- "	J.	18	12	31.2
61	" 9	"	"	"	18- "	J.T.	18	12	31.2
62	" 9	"	"	"	Heliom.	S.	18	12	31.3
63	" 9	"	"	"	6-inch	A.M.	18	12	31.4
64	" 9	"	"	"	7- "	H.	18	12	31.5
65	" 12	B.D. -4° 5757 (pr.)	6.8	"	18- "	J.	17	44	04.1
66	" 12	"	"	"	6- "	A.M.	17	44	04.1
67	" 12	" (f.)	"	"	18- "	J.	17	44	08.1
68	" 12	"	"	"	6- "	A.M.	17	44	08.1
69	" 19	δ Tauri	3.9	R.D.	7- "	H.	21	52	28.9
70	" 19	68 Tauri	4.2	D.B.	7- "	H.	22	34	49.6

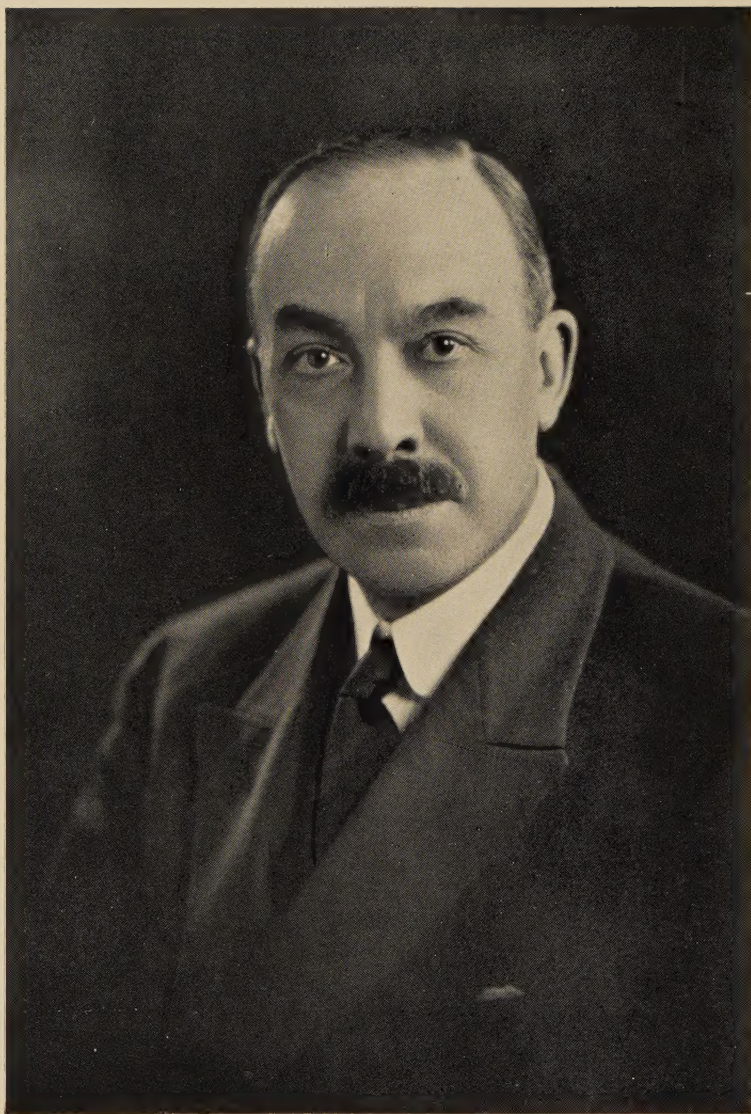
No.	Date 1940	Star	Mag.	Phase	Inst.	Obs.	U.T.		
							h	m	s
71	Oct. 20	68 Tauri.	4.2	R.D.	Heliom.	S.	0	01	31.7
72	" 20	"	"	"	7-inch	H.	0	01	32.0
73	" 22	71 Orionis	5.2	"	6- "	A.M.	2	27	33.4
74	" 22	"	"	"	7- "	S.	2	27	33.9
75	Nov. 2	B.D. -17° 4759	7.4	D.D.	6- "	A.M.	18	25	22.0
76	" 6	B.D. -12° 5890	7.1	"	6- "	A.M.	20	24	13.8
77	" 12	B.D. +5° 131	6.3	"	10- "	A.M.	0	09	31.9
78	" 12	"	"	"	6- "	S.	0	09	32.5
79	Dec. 5	B.D. -6° 5972	7.5	"	10- "	A.M.	20	03	47.3

Equations of Condition

No.	
1	$+1.0 = +12.2\Delta\alpha - .51\Delta\delta = +.85\Delta L - .53\Delta B$
2	$0.0 = -14.6 - .19 = -.87 - .49$
3	$+1.2 = -10.4 - .72 = -.42 - .91$
4-5	$+0.2 = +13.4 + .36 = +.99 + .17$
6-8	$+0.2 = +14.0 + .17 = +.97 + .26$
9	$+0.2 = -11.5 + .60 = -.85 + .53$
10-11	$-0.2 = +13.4 - .35 = +.98 - .21$
12	$-1.9 = +11.5 - .61 = +.90 - .43$
13-14	$0.0 = +10.6 - .69 = +.88 - .48$
15-17	$-2.2 = +11.6 - .62 = +.93 - .36$
18-20	$-0.8 = +10.3 - .72 = +.87 - .49$
21	$+1.0 = +3.2 - .97 = +.32 - .95$
22-23	$-0.2 = +13.8 - .28 = +.99 - .11$
24-25	$+0.2 = +12.4 + .54 = +.65 + .76$
26	$0.0 = +5.5 - .93 = +.51 - .86$
27	$+0.2 = -10.1 - .72 = -.80 - .60$
28	$+0.7 = -14.9 + .14 = -.89 + .45$
29	$+1.4 = +12.4 - .54 = +.96 - .29$
30	$0.0 = +14.2 - .12 = +1.00 - .03$
31-33	$-0.1 = +14.4 - .21 = +1.00 + .04$
34-36	$-1.6 = +13.8 + .35 = +.82 + .57$
37-38	$-0.1 = +9.9 - .73 = +.81 - .59$
39-40	$-1.1 = +12.3 + .53 = +.74 + .67$
41	$-1.7 = +13.8 + .27 = +.93 + .38$
42	$-1.3 = +10.0 + .72 = +.66 + .75$
43-45	$-1.1 = +13.4 - .36 = +.86 - .51$
46	$+0.3 = -10.8 - .65 = -.84 - .54$
47-48	$-1.0 = +7.2 + .86 = +.46 + .89$
49-50	$-1.3 = +13.8 + .25 = +.95 + .30$
51-53	$-1.4 = +4.2 + .95 = +.37 + .93$
54-55	$-2.3 = +12.9 + .42 = +.91 + .42$
56-59	$+1.7 = +10.0 - .73 = +.53 - .85$
60-64	$+0.2 = +13.6 - .34 = +.85 - .52$
65-68	$-1.2 = +4.4 + .96 = +.57 + .82$
69	$+0.2 = -6.5 - .89 = -.56 - .83$
70	$+4.0 = +13.3 + .37 = +.97 + .26$
71-72	$-0.3 = -14.2 - .14 = -1.00 - .02$
73-74	$+0.2 = -5.7 + .92 = -.43 + .90$
75	$+1.0 = +11.9 - .55 = +.86 - .52$
76	$-0.5 = +12.9 + .48 = +.96 + .27$
77-78	$-0.1 = +12.9 - .50 = +.67 - .74$
79	$-0.3 = +4.8 + .95 = +.58 + .81$

1. Through thick cloud.
4-5. Through thick passing cloud.

77-78. Bad definition.
79. Through cloud.



PROFESSOR H. C. PLUMMER, F.R.S.
President, 1939-41.